Written Exercises

Find an equation of the circle with the given center and radius.

A

1. (3, 0); 3
2. (0, −1); 1
3. (2, −5); 8
4. (−3, 1); 5
5. (0, 0); 12
6. (−4, −2); 10
7. (6, 1); $\sqrt{2}$
8. (−5, 3); $\frac{1}{6}$

Graph each equation. You may wish to check your graphs on a computer or a graphing calculator.

9. $x^2 + y^2 = 25$
10. $x^2 + y^2 = 4$
11. $(x - 4)^2 + (y - 5)^2 = 1$
12. $(x + 2)^2 + (y + 3)^2 = 81$
13. $(x - 3)^2 + y^2 = 36$
14. $x^2 + (y + 6)^2 = 4$

If the graph of the given equation is a circle, find its center and radius. If the equation has no graph, say so.

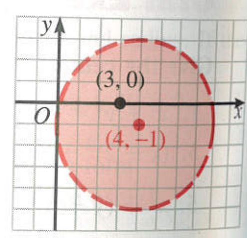
15. $x^2 + y^2 - 16 = 0$
16. $x^2 + y^2 - 81 = 0$
17. $x^2 + y^2 = -8y$
18. $x^2 + y^2 - 6x = 0$
19. $x^2 + y^2 - 4x + 2y - 4 = 0$
20. $x^2 + y^2 + 10x - 4y + 20 = 0$
21. $x^2 + y^2 + 8x + 2y + 18 = 0$
22. $x^2 + y^2 + 12x - 6y = 0$
23. $x^2 + y^2 + 3x - 4y = 0$
24. $x^2 + y^2 - 5y + 4 = 0$

Graph each inequality.

Sample $x^2 + y^2 - 8x + 2y + 1 < 0$

Solution Rewrite the inequality by completing the squares in x and y. The inequality becomes $(x - 4)^2 + (y + 1)^2 < 16$.

The graph of the equation $(x - 4)^2 + (y + 1)^2 = 16$ is the circle with center (4, −1) and radius 4. Sketch this circle using dashed lines to indicate that points on the circle do not satisfy the inequality. To determine whether the graph is the set of points inside or outside the circle, substitute a test point, say (3, 0), in the inequality. Since (3, 0) satisfies the inequality, shade the region inside the circle for the graph.



B

25. $x^2 + y^2 \geq 1$
26. $x^2 + y^2 < 4$
27. $x^2 + y^2 - 4y > 0$
28. $x^2 + y^2 \geq 2y$
29. $x^2 + y^2 + 6x - 6y + 9 \leq 0$
30. $x^2 + y^2 + 4x - 10y < 7$

Find the center and radius of each circle. (*Hint:* First divide both sides by the coefficient of the second-degree terms.)

31. $4x^2 + 4y^2 - 16x - 24y + 36 = 0$
32. $9x^2 + 9y^2 + 6x + 18y + 9 = 0$
33. $16x^2 + 16y^2 - 32x + 8y = 0$
34. $3x^2 + 3y^2 - 6x + 24y + 24 = 0$

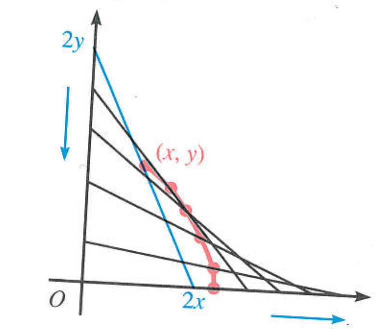
Find an equation of the circle described. (A sketch may be helpful.)

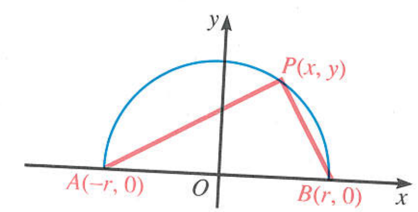
35. Center (0, 5); passes through (0, 0).
36. Center (−2, 0); passes through (2, 0).
37. A diameter has endpoints (2, 5) and (0, 3).
38. Center in quadrant two; radius 3; tangent to y-axis at (0, 4).
39. Center on line $y - 4 = 0$; tangent to x-axis at (−2, 0).
40. Center on line $x + y = 4$; tangent to both coordinate axes.
41. Center in quadrant four; tangent to the lines $x = 1$, $x = 9$, and $y = 0$.
42. Tangent to both coordinate axes and the line $x = -8$. (Two answers)

Graph each semicircle. Recall that $\sqrt{a} \geq 0$.

C

43. $y = \sqrt{25 - x^2}$
44. $y = -\sqrt{25 - x^2}$
45. $y = \sqrt{4x - x^2}$
46. $x = \sqrt{2y - y^2}$
47. A boat sails so that it is always twice as far from one buoy as from a second buoy 3 mi from the first one. Describe the path of the boat. (*Hint:* Introduce a coordinate system so that one buoy is at (0, 0) and the other is at (3, 0).)
48. A ladder 6 m long leaning against a wall slips to the ground. Describe the path followed by the midpoint of the ladder. (*Hint:* See the figure at the right.)
49. Find an equation of the circle of radius 4 that is tangent to both branches of the graph of $y = |x|$.
50. Use analytic geometry to prove that an angle inscribed in a semicircle is a right angle. (*Hint:* Let the semicircle be the top half of the circle $x^2 + y^2 = r^2$ and $P(x, y)$ be any point on the semicircle. Use the slopes of $\overline{PA}$ and $\overline{PB}$ to show that they are perpendicular.)





Challenge

Points with integer coordinates are called *lattice points*. For example, (0, 0) and (−2, 5) are lattice points, but $\left(\frac{1}{2}, 3\right)$ is not. The circle $x^2 + y^2 = 1$ passes through the 4 lattice points (1, 0), (−1, 0), (0, 1), and (0, −1). Find an equation of a circle that passes through exactly 3 lattice points.