Use the relationship $c^2 = a^2 - b^2$ to find the foci. Since $a^2 = 64$ and $b^2 = 16$, $c^2 = 48$ and $c = \sqrt{48} = 4\sqrt{3} \approx 6.9$. Thus the foci of the ellipse are $F_1(0, -4\sqrt{3})$ and $F_2(0, 4\sqrt{3})$. ***Answer***

Example 2 Find an equation of an ellipse having x-intercepts $\sqrt{2}$ and $-\sqrt{2}$ and y-intercepts 3 and -3.

Solution Since $3^2 > (\sqrt{2})^2$, the y^2-term has the larger denominator, and so the major axis is vertical. The center is $(0, 0)$, $a^2 = 9$, and $b^2 = 2$.

$\therefore$ an equation is $\dfrac{x^2}{2} + \dfrac{y^2}{9} = 1$. ***Answer***

Example 3 Find an equation of an ellipse having foci $(-3, 0)$ and $(3, 0)$ and sum of focal radii equal to 12.

Solution The center of the ellipse is $(0, 0)$. The distance from each focus to the center is 3, so $c = 3$. The sum of the focal radii is $2a$. Since $2a = 12$, $a = 6$ and $a^2 = 36$. So $b^2 = a^2 - c^2 = 36 - 9 = 27$. Since the foci are on the x-axis, the major axis is horizontal.

$\therefore$ an equation is $\dfrac{x^2}{36} + \dfrac{y^2}{27} = 1$. ***Answer***

Oral Exercises

Give the x- and y-intercepts of each ellipse and tell on which of the coordinate axes its foci lie.

1. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
2. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$
3. $x^2 + 9y^2 = 36$
4. $x^2 + 4y^2 = 16$
5. $3x^2 + y^2 = 9$
6. $2x^2 + 3y^2 = 6$

Written Exercises

Graph each ellipse and find its foci. You may wish to check your graphs on a computer or a graphing calculator.

A **1–6.** Use the equations in Oral Exercises 1–6.

7. $5x^2 + y^2 = 25$
8. $x^2 + 25y^2 = 100$
9. $3x^2 + y^2 = 12$
10. $2x^2 + y^2 = 8$
11. $4x^2 + 3y^2 = 48$
12. $5x^2 + 9y^2 = 45$
13. $x^2 + 9y^2 = 1$
14. $9x^2 + 4y^2 = 9$

Find an equation of an ellipse having the given intercepts.

15. x-intercepts: ± 5
y-intercepts: ± 2

16. x-intercepts: ± 3
y-intercepts: ± 4

17. x-intercepts: ± 2
y-intercepts: $\pm\sqrt{2}$

18. x-intercepts: $\pm\sqrt{6}$
y-intercepts: $\pm 2\sqrt{3}$

Find an equation of the ellipse having the given points as foci and the given number as sum of focal radii.

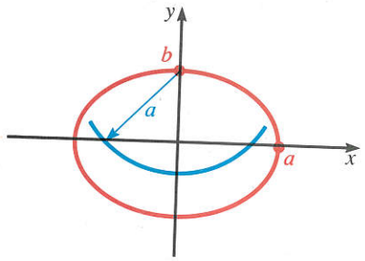
B **19.** $(-6, 0)$, $(6, 0)$; 18

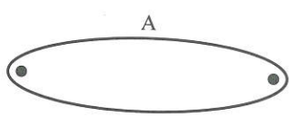
20. $(0, -5)$, $(0, 5)$; 20

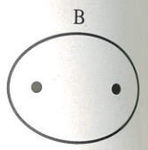
21. $(0, -4)$, $(0, 4)$; 24

22. $(-9, 0)$, $(9, 0)$; 30

23. Describe a way of constructing the foci of a given ellipse using a compass. Give a convincing argument to justify the method. (*Hint:* See the figure at the left below.)







24. An indicator of the shape of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is its *eccentricity* e, defined by $e = \frac{c}{a}$, where $c = \sqrt{a^2 - b^2}$.

a. Explain why is e a number between 0 and 1.
b. Which of the ellipses shown above has the greater eccentricity?

25. Suppose that $a = b$ in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

a. What is the value of c?
b. What is the value of e?
c. Where are the two foci?
d. What special ellipse will the graph be?

26. Describe the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$.

27. The arch of a bridge is in the form of half an ellipse, with the major axis horizontal. The span of the bridge (the length of the major axis) is 12 m and the height of the arch above the water at the center is 4 m. How high above the water is the arch at a point on the water 2 m from one of the ends of the arch?

Graph each inequality.

28. $4x^2 + 9y^2 < 36$

29. $4x^2 + y^2 \geq 16$

30. $5x^2 + 4y^2 \geq 20$

31. $3x^2 + 16y^2 \leq 48$

Graph each equation. Each graph is a semi-ellipse, since $\sqrt{a}$ is always nonnegative.

32. $y = 3\sqrt{1 - x^2}$

33. $y = \frac{1}{2}\sqrt{4 - x^2}$

C **34.** For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci at $(-c, 0)$ and $(c, 0)$, show that $b^2 = a^2 - c^2$.

35. Use the definition to find an equation of the ellipse with foci at $(-c, 0)$ and $(c, 0)$ and sum of focal radii equal to $2a$. At the appropriate point let $b^2 = a^2 - c^2$ to simplify the equation.

Career Note / Computer Graphics Artist

In movies and on television you have probably seen believable pictures of things that do not exist and cannot be photographed. These pictures are the product of computer graphics. The computer graphics artist gives the computer a complete description of each object that is to be in a picture. The object's shape may be built up from flat polygons or from geometric solids such as spheres. The location of each object in the scene is given in three-dimensional coordinates so that objects farther from the viewer will appear smaller.

A computer program compares the boundaries of objects and decides which surfaces are visible to the viewer. In order to give realistic texture to images, the graphics artist uses formulas describing how different materials reflect light. Even using powerful computers, it may take the computer graphics artist hours to make one second of movie film.

The computer has become a valuable tool for graphic artists in many other fields. For example, computer-generated shapes and color patterns are now used in the design of magazine illustrations, posters, packaging, and textiles.