Oral Exercises

Give the intercepts of the hyperbola and the equations of its asymptotes. Tell on which of the coordinate axes its foci lie.

1. $\frac{x^2}{25} - \frac{y^2}{16} = 1$
2. $\frac{y^2}{1} - \frac{x^2}{9} = 1$
3. $x^2 - 25y^2 + 25 = 0$
4. $4x^2 - y^2 = 16$
5. $25x^2 - 4y^2 = 100$
6. $4x^2 - 9y^2 + 36 = 0$

Describe the graph of each equation.

7. $xy = k$, when $k = 0$
8. $y = \frac{1}{x}$
9. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Written Exercises

In Exercises 1–12, (a) graph each hyperbola, showing its asymptotes as dashed lines; (b) find the coordinates of the foci. You may wish to check your graphs on a computer or a graphing calculator.

A

1–6. Use the equations in Oral Exercises 1–6.

7. $x^2 = 9y^2 - 81$
8. $y^2 = 5x^2 + 25$
9. $75x^2 - 100y^2 = 7500$
10. $25x^2 - 144y^2 = 3600$
11. $4x^2 - y^2 + 1 = 0$
12. $16x^2 - 4y^2 + 64 = 0$

Find an equation of the hyperbola described.

13. Foci $(0, -8)$ and $(0, 8)$; difference of focal radii 10.
14. Foci $(-4, 0)$ and $(4, 0)$; difference of focal radii 4.
15. Asymptotes $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$; foci $(0, -\sqrt{13})$ and $(0, \sqrt{13})$.
16. Asymptotes $y = \frac{\sqrt{2}}{2}x$ and $y = -\frac{\sqrt{2}}{2}x$; foci $(0, -\sqrt{6})$ and $(0, \sqrt{6})$.

B

17. Asymptotes $y = 3x$ and $y = -3x$; y-intercepts 3 and -3.
18. Asymptotes $y = x$ and $y = -x$; foci $(-4, 0)$ and $(4, 0)$.

Graph each inequality.

19. $y^2 - x^2 > 4$
20. $y^2 \le x^2 - 4$
21. $4x^2 \le y^2 + 16$
22. $9x^2 > 4y^2 - 36$

The hyperbolas $\frac{x^2}{p^2} - \frac{y^2}{q^2} = 1$ and $\frac{y^2}{q^2} - \frac{x^2}{p^2} = 1$ are *conjugates* of each other. Graph the following conjugate hyperbolas on the same coordinate axes.

23. $\frac{x^2}{9} - \frac{y^2}{4} = 1$; $\frac{y^2}{4} - \frac{x^2}{9} = 1$
24. $\frac{x^2}{1} - \frac{y^2}{4} = 1$; $\frac{y^2}{4} - \frac{x^2}{1} = 1$

Graph each equation. Each graph is half of a hyperbola, since $\sqrt{a}$ is nonnegative.

25. $y = \sqrt{x^2 + 16}$
26. $y = \sqrt{x^2 - 1}$
27. $y = \sqrt{x^2 - 16}$
28. $y = \sqrt{x^2 + 1}$
29. The statement "traveling 200 miles at x mi/h for y hours" can be described by the equation $xy = 200$. Consider the restrictions on x and y and then graph this equation. What does the graph tell you about the relationship between x and y?

Use the definition to find an equation of the hyperbola having the given points as foci and the given number as difference of focal radii.

30. $(0, -5)$, $(0, 5)$; 4
31. $(-3, 0)$, $(3, 0)$; 2

C

32. $(-c, 0)$, $(c, 0)$; $2a$
33. $(0, -c)$, $(0, c)$; $2a$
34. (a, a), $(-a, -a)$; $2a$
35. $(-a, a)$, $(a, -a)$; $2a$

Mixed Review Exercises

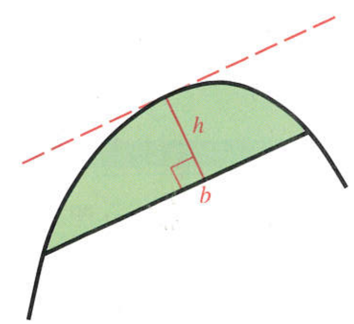
Graph each equation.

1. $x^2 + 4y^2 = 16$
2. $x^2 + y^2 - 2x + 4y + 1 = 0$
3. $3x - 4y = 6$
4. $2x^2 - 4x + y + 5 = 0$

Draw the graph of each relation and tell whether or not it is a function.

5. $\{(x, y): y = 2x\}$
6. $\{(x, y): x = 2y\}$
7. $\{(x, y): x = y^2\}$
8. $\{(x, y): y = x^2\}$
9. $\{(x, y): y = |x|\}$
10. $\{(x, y): x = |y|\}$

Historical Note / *The Area of a Parabolic Section*



About 240 B.C. the great Greek mathematician Archimedes discovered a formula for the area of any section of a parabola. By summing the areas of a sequence of smaller and smaller triangles that fill the section more and more completely, Archimedes was able to show that the area of any section is $\frac{2}{3}bh$, where b is the length of the base (the chord that cuts off the section) and h is the height (the length of a perpendicular segment to the base from the point on the parabola where a tangent parallel to the base touches the parabola). Therefore, the area of the section is $\frac{4}{3}$ as large as the area of the largest triangle that can be inscribed in it.