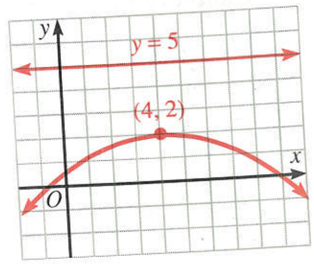
Example 4 Find an equation of the parabola that has vertex (4, 2) and directrix $y = 5$.

Solution The distance from the vertex to the directrix is 3, so $|c| = 3$. Since the directrix is above the vertex, the parabola opens downward. Therefore the squared term is the term with x, and c is negative. If $c = -3$, then $a = -\frac{1}{12}$. Thus the equation is

$$y - 2 = -\frac{1}{12}(x - 4)^2. \quad \textit{Answer}$$



Oral Exercises

In the following exercises, V denotes the vertex of a parabola, F the focus, and D the directrix. Two of these are given. Find the third.

1. $V(0, 0)$, $F(-3, 0)$
2. $V(0, 0)$, D: $x = -2$
3. $F(4, 6)$, D: $x = 0$
4. D: $y = -1$, $F(4, -4)$
5. D: $y = -2$, $V(3, -1)$
6. $F(-2, 5)$, $V(-2, 1)$

For each parabola, find (a) $V(h, k)$, (b) $|c|$, the distance between its vertex and focus, and (c) the direction in which it opens (up, down, left, right).

7. $y = \frac{1}{8}x^2$
8. $x = -\frac{1}{12}y^2$
9. $x = -(y - 2)^2$
10. $y = (x + 5)^2$
11. $x + 3 = \frac{1}{4}(y - 1)^2$
12. $y + 4 = -\frac{1}{36}(x - 4)^2$

Written Exercises

In the following exercises, V denotes the vertex of a parabola, F the focus, and D the directrix. Two of these are given. Find the third.

A

1. $V(4, 2)$, D: $y = -3$
2. D: $y = 2$, $V(2, 4)$
3. $V(0, 2)$, $F(0, 0)$
4. $F(-3, -1)$, $V(1, -1)$
5. $F(1, -2)$, $V(1, -5)$
6. D: $x = -2$, $F(2, 0)$

Find an equation of the parabola described. Then graph the parabola.

7. Focus (0, 0); directrix $y = 4$
8. Focus (0, 0); directrix $x = 4$
9. Vertex (0, 0); focus (0, −4)
10. Vertex (0, 0); focus (3, 0)
11. Vertex (0, 0); directrix $x + 1 = 0$
12. Vertex (0, 0); directrix $y = -4$
13. Focus (0, 2); directrix $x = 2$
14. Focus (−2, 0); directrix $y = 3$
15. Focus (3, 4); vertex (3, 2)
16. Focus (−2, 1); vertex (−3, 1)

Find the vertex, focus, directrix, and axis of symmetry of each parabola. Then graph the parabola. You may wish to check your graphs on a computer or a graphing calculator.

17. $y = \frac{1}{6}x^2$

18. $6x + y^2 = 0$

19. $4x = y^2 - 4y$

20. $x^2 = y + 2x$

B **21.** $x^2 + 8y + 4x - 4 = 0$

22. $y^2 + 6y + 8x - 7 = 0$

23. $y^2 - 8x - 6y - 3 = 0$

24. $x^2 - 6x + 10y - 1 = 0$

25. $x^2 + 10x - 2y + 21 = 0$

26. $y^2 + 3x - 2y - 11 = 0$

Graph each inequality.

27. $y < (x - 3)^2$

28. $y \geq 2(x + 1)^2$

29. $y + 4 \geq (x + 2)^2$

30. $x - 11 < y^2 + 6y$

Graph each equation. Each graph is half of a parabola, since $\sqrt{a} \geq 0$.

31. $y = \sqrt{x}$

32. $y = \sqrt{x - 2}$

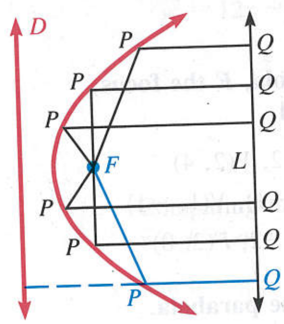
33. $y = -\sqrt{1 - x}$

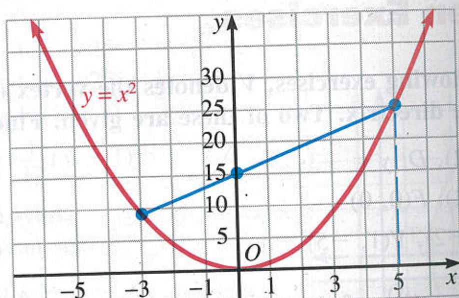
34. $y = -\sqrt{3 - x}$

C **35.** The line through the focus of a parabola perpendicular to its axis intersects the parabola in two points, P and Q. ($\overline{PQ}$ is called the *latus rectum* of the parabola.) Explain why the length of $\overline{PQ}$ is twice the distance from the focus to the directrix.

36. A parabola having vertex (h, k) and a vertical axis has as focus the point $(h, k + c)$ and as directrix the line $y = k - c$. Show that an equation of the parabola is $y - k = \frac{1}{4c}(x - h)^2$.

37. F is the focus of the parabolic arc shown below. Line L is parallel to the directrix, D. Explain why the sum $FP + PQ$ is the same for all positions of P. (*Hint:* Extend $\overline{PQ}$ so that it intersects the directrix.)





38. A *multiplication machine:* Draw the parabola $y = x^2$. Given any two positive numbers a and b, locate the points of the parabola having the x-coordinates $-a$ and b. Explain why the line joining these points has y-intercept ab. (*Hint:* Find an equation of the line through $(-a, a^2)$ and (b, b^2). Use the diagram above as a model.)