

Exercises 19 and 20 lead to cubic equations. Use the methods of Lesson 8-7 to find their real roots.

19.  $y^2 = x + 7$   
 $xy = 6$

20.  $y = x^2 - 1$   
 $xy + 6 = 0$

Find the square roots of each complex number.

**Sample**  $12 + 16i$

**Solution**

Let a square root of  $12 + 16i$  be  $x + yi$ , where  $x$  and  $y$  are real. Then  $(x + yi)^2 = 12 + 16i$ , or  $(x^2 - y^2) + 2xyi = 12 + 16i$ . Equating the real and imaginary parts gives this system:

$$\begin{aligned} x^2 - y^2 &= 12 \\ 2xy &= 16 \end{aligned}$$

This system is equivalent to the one in Example 2, page 440, and thus has the solutions  $(4, 2)$  and  $(-4, -2)$ . Therefore, the square roots of  $12 + 16i$  are  $4 + 2i$  and  $-4 - 2i$ .

**C** 21.  $3 + 4i$   
24.  $-5 - 12i$

22.  $7 - 24i$   
25.  $-7 + 24i$

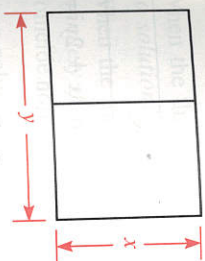
23.  $5 + 12i$   
26.  $12 - 16i$

## Problems

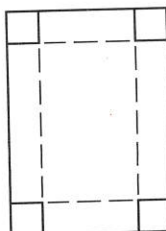
Solve.

- A**
- The sum of two numbers is 16, and the sum of their squares is 146. Find the numbers.
  - The product of two numbers is 1, and the difference of their squares is  $\frac{15}{4}$ . Find the numbers.
  - The fence around a rectangular piece of property is 156 m long. If the area of the property is  $1505 \text{ m}^2$ , find the dimensions of the property.
  - An ellipse with center at the origin and horizontal major axis is to fit snugly inside a rectangle that has its longer sides horizontal. The area of the rectangle is 12 square units, and the perimeter is 14 units. Find an equation for the ellipse.
  - Find the dimensions of a rectangle having perimeter 34 ft and a diagonal of length 13 ft.
  - Find the length of the legs of a right triangle having perimeter 56 m if the hypotenuse is 25 m.
  - The product of a two-digit number and its tens digit is 285. The units digit is two more than the tens digit. Find the original number.
  - Find the dimensions of a rectangle that has area 10 and a diagonal of length 5. Leave your answer in terms of radicals.

- B**
9. A rectangular plot of land having area  $1350 \text{ m}^2$  is to be enclosed and divided into two parts, as shown. Find the dimensions of the plot if the total length of fencing used is 180 m.



Ex. 9



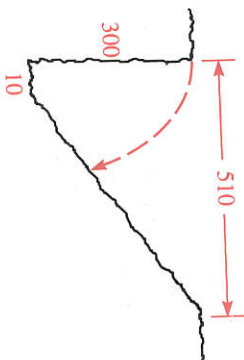
Ex. 10

10. Four squares, each with sides 4 cm long, are cut from the corners of a rectangular piece of cardboard having area  $560 \text{ cm}^2$ . The flaps are then bent up to form an open-topped box having volume  $960 \text{ cm}^3$ . Find the dimensions of the original piece of cardboard.
11. Two people part company and walk along perpendicular paths. One person walks 1 km/h faster than the other. They are 6 km apart after one hour. Find the rate at which each person walks. Give your answers to the nearest tenth.
12. A 20 m ladder and a 15 m ladder were leaned against a building. The bottom of the longer ladder was 7 m farther from the building than the bottom of the shorter ladder, but both ladders reached the same distance up the building. Find the distance.

- C**
13. Find the point on the circle with equation  $x^2 + y^2 = 1$  that is closest to the point  $(4, 3)$ .

14. From the top of a vertical canyon wall 300 m high, a person throws a rock with a speed of 20 m/s toward the top of the opposite side of the canyon, which is at the same elevation, but is 510 m away. According to the laws of physics, the rock follows a path described approximately by the equation

$$y = 300 - \frac{x^2}{80}$$



where  $y$  is the rock's height in meters above the canyon floor and  $x$  is the rock's horizontal distance in meters to the right of the vertical canyon wall. The opposite side of the canyon is steeply sloping and becomes level at a point 10 m from the base of the vertical wall. When the rock strikes the sloping side of the canyon, how far is it from where it was thrown? Express your answer rounded to the nearest tenth of a meter. (*Hint:* Introduce a coordinate system whose origin is the base of the vertical wall. Write an equation for the sloping side, and find a common solution of this equation and the equation of the rock's path.)