Written Exercises

By sketching graphs, find the *number* of real solutions the system has.

A

1. $4x^2 + 9y^2 = 36$
$2x + 3y = 6$

2. $16x^2 + 4y^2 = 64$
$3x + 4y = 12$

3. $x^2 - 9y = 0$
$x - 2y = 2$

4. $4x^2 + 25y^2 = 100$
$x^2 + y^2 = 64$

5. $x^2 - y^2 = 4$
$x + y = 4$

6. $y^2 - x^2 = 4$
$y = x^2 - 3$

7. $x^2 + y^2 = 25$
$xy = 10$

8. $9x^2 + 25y^2 = 225$
$y^2 = 2x + 4$

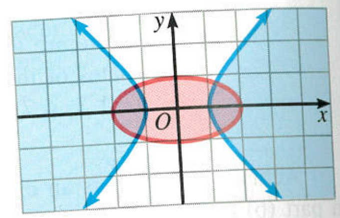
9. $x^2 - 4y^2 = 4$
$6y^2 - x = 2$

10–15. Estimate the real solutions (if any) of the systems of Exercises 4–9 to the nearest half unit.

Graph each system of inequalities.

Sample $x^2 + 4y^2 \leq 4$
$x^2 \geq y^2 + 1$

Solution



B

16. $y \geq x + 3$
$y \leq x^2$

17. $x^2 + 4y^2 \leq 16$
$x^2 \leq y^2 + 4$

18. $4x^2 + y^2 \leq 16$
$x^2 + 4y^2 \leq 16$

19. $x \geq -y^2$
$x^2 + y^2 \leq 49$

C

20. Use parts (a), (b), and (c) below to show that no isosceles triangle has perimeter 4 and area 1.

a. Write equations for the area and perimeter.
(*Hint:* Let base = $2x$ and height = y.)

b. Simplify the perimeter equation to eliminate radicals.

c. Graph the resulting system and use the graph to reach the required conclusion.

Mixed Review Exercises

Find an equation for each figure described.

1. Parabola with focus (2, 3) and directrix $y = -1$.
2. Ellipse with x-intercepts $\pm\sqrt{6}$ and y-intercepts ± 2.
3. Hyperbola with foci (0, −3) and (0, 3) and difference of focal radii 4.
4. The circle having center (−3, 4) and passing through the origin.

Find the unique solution for each system.

5. $y = 3x + 2$
$2x - y = -3$

6. $x - 3y = 5$
$4x + y = 7$

7. $5x + 2y = 9$
$4x - 3y = 21$

If $x = \sqrt{5}$:

$$(\sqrt{5})^2 + 2y^2 = 23$$
$$2y^2 = 18$$
$$y^2 = 9$$
$$y = 3 \quad \text{or} \quad y = -3$$

If $x = -\sqrt{5}$:

$$(-\sqrt{5})^2 + 2y^2 = 23$$
$$2y^2 = 18$$
$$y^2 = 9$$
$$y = 3 \quad \text{or} \quad y = -3$$

$\therefore$ the solution set is $\{(\sqrt{5}, 3), (\sqrt{5}, -3), (-\sqrt{5}, 3), (-\sqrt{5}, -3)\}$.

As the preceding examples show, substitution is usually the more appropriate method for solving a system consisting of a linear and a quadratic equation. When a system's equations are both quadratic, either the substitution or the linear-combination method may be used.

Oral Exercises

Which of the given ordered pairs are solutions of the system?

1. $xy + 5 = 0$, $2x + y + 3 = 0$ — $(5, -1), (-5, 1), (1, -5), (-1, 5)$
2. $x^2 + y^2 = 20$, $x + y + 2 = 0$ — $(4, 2), (-4, 2), (2, 4), (-2, 4)$
3. $x^2 + 4y^2 = 25$, $y^2 - x = 1$ — $(3, 2), (-3, 2), (-3, -2), (3, -2)$

Written Exercises

Find the real solutions, if any, of each system. You may wish to check your answers visually on a computer or a graphing calculator.

A

1. $x^2 - y = 5$; $2x + y = 3$
2. $x = y^2 - 9$; $x - 4y - 12 = 0$
3. $y = x^2$; $x^2 + y^2 = 12$
4. $y^2 = 2x$; $x^2 + y^2 = 8$
5. $x^2 - y^2 = 15$; $x + y = 1$
6. $x^2 + y^2 = 61$; $x - 2y + 7 = 0$
7. $xy = 8$; $x + y = 6$
8. $xy + 6 = 0$; $x - y = 5$
9. $2y^2 + 3x = 33$; $x + 4y + 7 = 0$
10. $4x^2 - y^2 + 12 = 0$; $x + y = 3$
11. $x^2 + y^2 = 25$; $x^2 - y^2 = 7$
12. $8x^2 + y^2 = 25$; $8x^2 - y^2 = 39$

B

13. $2x^2 - 3y^2 = 30$; $x^2 + y^2 = 25$
14. $x^2 + 2y^2 = 12$; $3x^2 - y^2 = 8$
15. $5x^2 + 3y^2 = 7$; $3x^2 - 7y^2 = 13$
16. $9x^2 + 9y^2 = 1$; $x = y^2 + 1$
17. $x^2 + y^2 = 13$; $xy + 6 = 0$
18. $2x^2 - y^2 = 7$; $xy = 3$