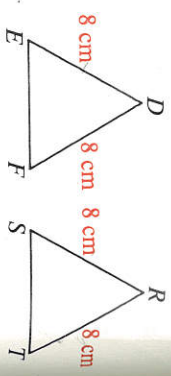
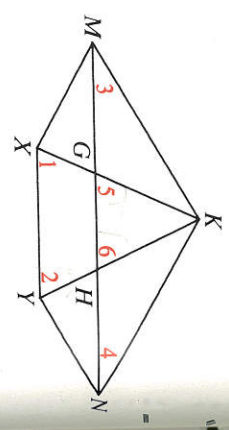


4.  $\triangle DEF$  and  $\triangle RST$  are isosceles, with legs 8 cm long. Name all pairs of angles that are congruent.



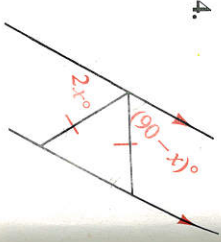
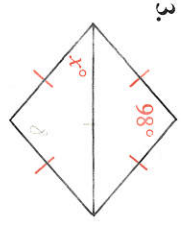
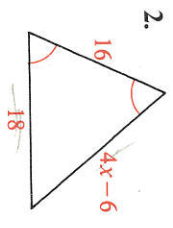
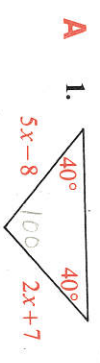
5. If  $\angle 1 \cong \angle 2$ , then  $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$ .  
 6. If  $m\angle 3 = m\angle 4$ , then  $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ .  
 7. If  $\angle 5 \cong \angle 6$ , then  $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$ .  
 8. True or false?  $MK = NK$  if and only if  $m\angle 3 = m\angle 4$ .



9. Prove Theorem 3-1.  
 10. Explain how Corollary 1 follows from Theorem 3-1.  
 11. Explain how Corollary 2 follows from Corollary 1.  
 12. What is the converse of Corollary 1? Is this converse true?  
 13. Use the diagram for Theorem 3-1 to explain why Corollary 3 is true.

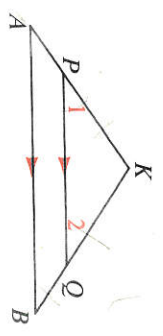
**Written Exercises**

Find the value of  $x$ .



5. Supply the missing statements and reasons.

Given:  $\overline{KA} \cong \overline{KB}$   
 $\overline{PQ} \parallel \overline{AB}$   
 Prove:  $\overline{KP} \cong \overline{KQ}$



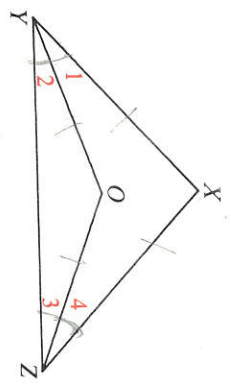
Statements

Reasons

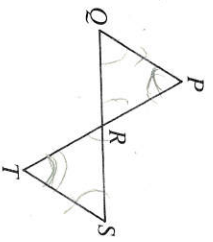
- |  |                                     |
|--|-------------------------------------|
| 1. $\overline{KA} \cong \overline{KB}$                     | 1. $\underline{\hspace{1cm}}$       |
| 2. $\angle A \cong \angle B$                               | 2. $\underline{\hspace{1cm}}$       |
| 3. $\overline{PQ} \parallel \overline{AB}$                 | 3. $\underline{\hspace{1cm}}$       |
| 4. $\angle KPQ \cong \angle 1$ ; $\angle B \cong \angle 2$ | 4. $\underline{\hspace{1cm}}$       |
| 5. $\underline{\hspace{1cm}}$                              | 5. Transitive Prop. (Steps 2 and 4) |
| 6. $\overline{KP} \cong \overline{KQ}$                     | 6. $\underline{\hspace{1cm}}$       |

**Write proofs in two-column form.**

6. Given:  $\angle R \cong \angle S$ ;  $\angle TUV \cong \angle TVU$   
 Prove:  $\overline{RU} \cong \overline{SV}$   
 7. Given:  $\overline{TU} \cong \overline{TV}$ ;  $\overline{UV} \parallel \overline{RS}$   
 Prove:  $\angle R \cong \angle S$   
 8. Given:  $\overline{XY} \cong \overline{XZ}$ ;  $\overline{OY} \cong \overline{OZ}$   
 Prove:  $m\angle 1 = m\angle 4$   
 9. Given:  $\overline{XY} \cong \overline{XZ}$ ;  
 $\overline{YO}$  bisects  $\angle XYZ$ ;  
 $\overline{ZO}$  bisects  $\angle XZY$ .  
 Prove:  $\overline{YO} \cong \overline{ZO}$

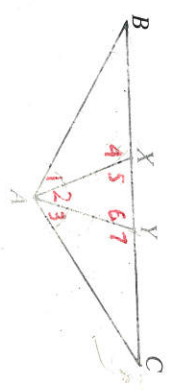


10. Given:  $\overline{PQ} \cong \overline{PR}$ ;  $\overline{TR} \cong \overline{TS}$   
 Which one(s) of the following *must* be true?  
 (1)  $\angle T \cong \angle P$  (2)  $\overline{ST} \cong \overline{QP}$  (3)  $\overline{ST} \parallel \overline{QP}$   
 11. Given:  $\angle S \cong \angle T$ ;  $\overline{ST} \parallel \overline{QP}$   
 Which one(s) of the following *must* be true?  
 (1)  $\angle P \cong \angle Q$  (2)  $\overline{PR} \cong \overline{QR}$   
 (3) R is the midpoint of  $\overline{PT}$ .

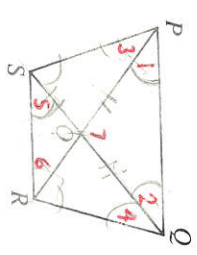


**Write proofs in two-column form.**

12. Given:  $\overline{AB} \cong \overline{AC}$ ;  $\overline{AX}$  and  $\overline{AY}$  trisect  $\angle BAC$ .  
 (This means  $\angle 1 \cong \angle 2 \cong \angle 3$ .)  
 Prove:  $\overline{AX} \cong \overline{AY}$   
 13. Given:  $\angle 4 \cong \angle 7$ ;  $\angle 1 \cong \angle 3$   
 Prove:  $\triangle ABC$  is isosceles.



14. Given:  $\angle 1 \cong \angle 2$ ;  $\angle 3 \cong \angle 4$   
 Prove:  $\angle 5 \cong \angle 6$   
 15. Given:  $\overline{PO} \cong \overline{QO}$ ;  $\overline{RO} \cong \overline{SO}$   
 a. If you are also given that  $m\angle 1 = 40$ , find the measures of  $\angle 2$ ,  $\angle 7$ ,  $\angle 5$ , and  $\angle 6$ . Then decide whether  $\overline{PQ}$  must be parallel to  $\overline{SR}$ .  
 b. If  $m\angle 1 = x$ , find the measures of  $\angle 2$ ,  $\angle 7$ ,  $\angle 5$ , and  $\angle 6$ . Is  $\overline{PQ} \parallel \overline{SR}$ ?



16. Draw an isosceles  $\triangle ABC$  whose vertex angle,  $\angle A$ , has measure 80.  
 a. Draw  $\overline{AX}$ , the bisector of an exterior angle at A. Is  $\overline{AX} \parallel \overline{BC}$ ? Explain.  
 b. Would your answer change if the measure of  $\angle A$  changed?

17. a. If  $m\angle 1 = 20$ , then  $m\angle 3 = \underline{\hspace{1cm}}$ ,  $m\angle 4 = \underline{\hspace{1cm}}$ , and  $m\angle 5 = \underline{\hspace{1cm}}$ .  
 b. If  $m\angle 1 = x$ , then  $m\angle 3 = \underline{\hspace{1cm}}$ ,  $m\angle 4 = \underline{\hspace{1cm}}$ , and  $m\angle 5 = \underline{\hspace{1cm}}$ .



# 3-5 Other Methods for Proving Triangles Congruent

The SSS, SAS, and ASA Postulates are used to prove two triangles congruent. In this section we will learn how to use other methods to prove triangles congruent.

## Theorem 3-3 AAS

If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent.

Given:  $\triangle ABC$  and  $\triangle DEF$ ,  $\angle C \cong \angle F$ ,  $\overline{AC} \cong \overline{DF}$   
 Prove:  $\triangle ABC \cong \triangle DEF$

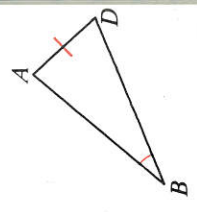
**Plan for Proof:** From the two angles of the other, construct a third angle to reach the desired conclusion.

Do you see overlapping triangles? Sometimes you can use overlapping triangles to prove two triangles congruent. For example, suppose you are given:

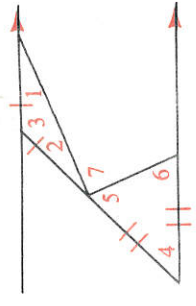
Given:  $\overline{AD} \cong \overline{AE}$ ,  $\angle B \cong \angle C$

Prove:  $\triangle ABD \cong \triangle ACE$

You may find it helpful to redraw the triangles. Now you can see that the two triangles are congruent by the AAS Theorem.



19. a. If  $m\angle 1 = 23$ , then  $m\angle 7 = ?$ .  
 b. If  $m\angle 1 = x$ , then  $m\angle 7 = ?$ .



Solve for  $x$  and  $y$ .

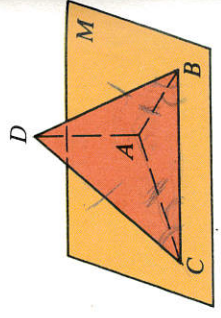
20.  $\triangle ABC$  is equilateral,  $AB = 4x - y$ ,  $BC = 2x + 3y$ , and  $AC = 7$ .

21.  $\triangle DEF$  is equilateral,  $m\angle D = x + y$ , and  $m\angle E = 2x - y$ .

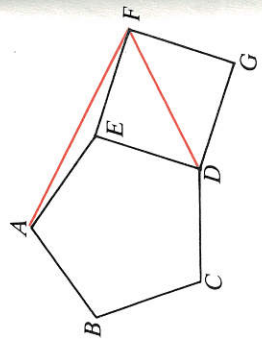
22. In  $\triangle JKL$ ,  $\overline{JK} \cong \overline{KL}$ ,  $m\angle J = 2x - y$ ,  $m\angle K = 2x + 2y$ , and  $m\angle L = x + 2y$ .

23. Given:  $\triangle ABC$  in plane  $M$ ;  $D$  not in plane  $M$ ;  
 $\angle ACB \cong \angle ABC$ ;  $\angle DCB \cong \angle DBC$

Name a pair of congruent triangles.  
 Prove your answer correct.



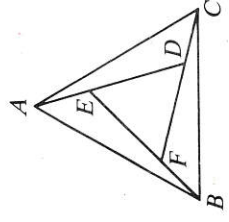
24.  $ABCDE$  is a regular pentagon and  $DEFG$  is a square. Find the measures of  $\angle EAF$  and  $\angle AFD$ .



**C** 25. Draw an isosceles triangle and then join the midpoints of its sides to form another triangle. What can you deduce about this second triangle? Explain.

26. Given:  $\triangle ABC$  is equilateral;  
 $\angle CAD \cong \angle ABE \cong \angle BCF$

Prove something interesting about  $\triangle DEF$ .



28. a. The figure on the left is a regular pentagon. Find the measures of  $\angle 1$ ,  $\angle 2$ , and  $\angle 3$ .

b. The figure on the right is a regular hexagon. Find the measures of  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ , and  $\angle 4$ .

