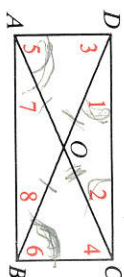**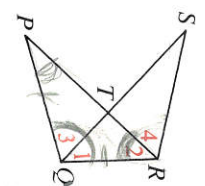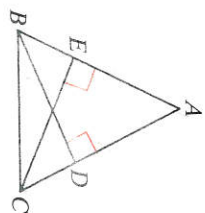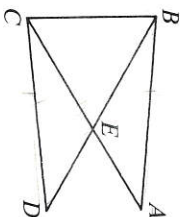For each diagram, name a pair of overlapping congruent triangles. Tell whether the triangles are congruent by the SSS, SAS, ASA, AAS, or HL method.**

11. Given: $AB = DC$;
$AC = DB$

12. Given: $\angle ABC \cong \angle ACB$

13. Given: $\angle 1 \cong \angle 2$;
$\angle 3 \cong \angle 4$

14. Given: $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$
Prove as much as you can about the diagram.

15. To prove that right triangles are congruent, some geometry books also use the methods stated below. Tell which of our methods (SSS, SAS, ASA, AAS, or HL) could be used instead of each method listed.

a. **Leg-Leg Method (LL)** If two legs of one right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.

b. **Hypotenuse-Acute Angle Method (HA)** If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

c. **Leg-Acute Angle Method (LA)** If a leg and an acute angle of one right triangle are congruent to the corresponding parts in another right triangle, then the triangles are congruent.
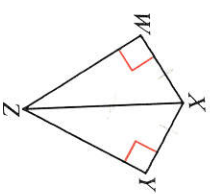
Written Exercises

Supply the missing statements and reasons.

A 1. Given: $\angle W$ and $\angle Y$ are rt. $\triangle$s;
$\overline{WX} \cong \overline{YX}$
Prove: $\overline{WZ} \cong \overline{YZ}$

Proof:

Statements	Reasons
1. $\angle W$ and $\angle Y$ are rt. $\triangle$s.	1. ?
2. $\triangle XWZ$ and $\triangle XYZ$ are rt. $\triangle$s.	2. ?
3. $\overline{WX} \cong \overline{YX}$	3. ?
4. ?	4. Reflexive Prop.
5. $\triangle WXZ \cong$?	5. ?
6. ?	6. ?

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2. Given: $\overline{KL} \perp \overline{LA}$; $\overline{KJ} \perp \overline{JA}$;
$\overrightarrow{AK}$ bisects $\angle LAJ$.
Prove: $\overline{LK} \cong \overline{JK}$

Proof:
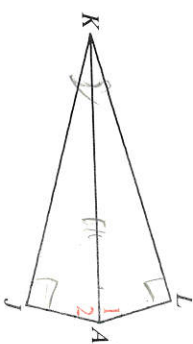
Statements	Reasons
1. $\overline{KL} \perp \overline{LA}$; $\overline{KJ} \perp \overline{JA}$	1. ?
2. $m\angle L = 90 = m\angle$?	2. Def. of $\perp$ lines and def. of rt. $\angle$
3. $\overrightarrow{AK}$ bisects $\angle LAJ$.	3. ?
4. ? $\cong$?	4. Def. of $\angle$ bisector
5. $\overline{KA} \cong$?	5. ?
6. $\triangle LKA \cong$?	6. ?
7. ?	7. ?

In Exercises 3 and 4 write proofs in two-column form.

3. Given: $\overline{EF} \perp \overline{EG}$; $\overline{HG} \perp \overline{EG}$;
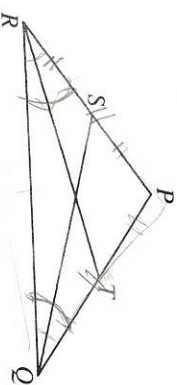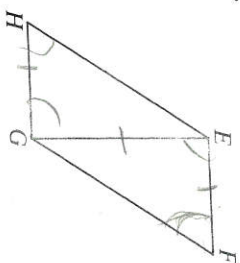$\overline{EH} \cong \overline{GF}$
Prove: $\angle H \cong \angle F$

4. Given: $\overline{EF} \parallel \overline{HG}$;
$\angle H \cong \angle F$
Prove: $\overline{HE} \cong \overline{FG}$

5. Given: $\overline{PR} \cong \overline{PQ}$;
$\overline{SR} \cong \overline{TQ}$
Prove: $\overline{QS} \cong \overline{RT}$

Plan for Proof: $\overline{QS}$ and $\overline{RT}$ are corresponding parts of $\triangle PQS$ and $\triangle PRT$ and also of $\triangle RQS$ and $\triangle QRT$. The second set of triangles is easier to prove congruent than the first set.

Proof:

Statements	Reasons
1. $\overline{PR} \cong \overline{PQ}$	1. ?
2. $\angle PQR \cong$?	2. If 2 sides of a $\triangle$ are $\cong$, then ?.
3. $\overline{SR} \cong \overline{TQ}$	3. ?
4. $\overline{RQ} \cong \overline{RQ}$	4. ?
5. $\triangle RQS \cong$?	5. ?
6. $\overline{QS} \cong \overline{RT}$	6. ?

6. Given: $m\angle 1 = m\angle 2;$
$\qquad m\angle 3 = m\angle 4$

Prove: $\overline{MJ} \cong \overline{NL}$

■ **Plan for Proof:** $\overline{MJ}$ and $\overline{NL}$ are corresponding parts of $\triangle MJN$ and $\triangle NLM$. Try to prove these triangles congruent.
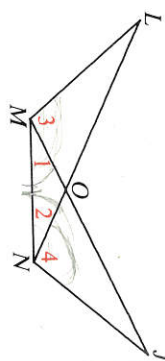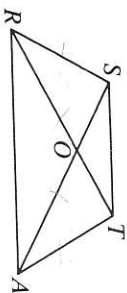
Proof:

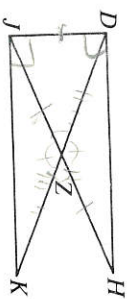| Statements	Reasons
1. $m\angle 1 = m\angle 2;$ $\quad m\angle 3 = m\angle 4$	1. $\underline{\ ?\ }$
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	2. $\underline{\ ?\ }$
3. $m\angle 1 + m\angle 3 = m\angle LMN;$ $\quad m\angle 2 + \underline{\ ?\ } = \underline{\ ?\ }$	3. $\underline{\ ?\ }$
4. $m\angle LMN = m\angle JNM$	4. $\underline{\ ?\ }$
5. $\underline{\ ?\ }$	5. Reflexive Prop.
6. $\triangle MJN \cong \underline{\ ?\ }$	6. $\underline{\ ?\ }$
7. $\underline{\ ?\ }$	7. $\underline{\ ?\ }$

7. Given: $\overline{RT} \cong \overline{AS};$
$\qquad \overline{RS} \cong \overline{AT}$
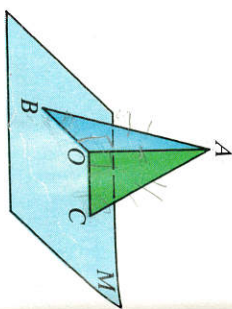
Prove: $\angle TSA \cong \angle STR$

8. Given: $\overline{DH} \perp \overline{DJ};\ \overline{JK} \perp \overline{DJ};$
$\qquad \overline{JH} \cong \overline{DK}$

Prove: $\angle H \cong \angle K$

9. Given: $\overline{AO} \perp$ plane M
State the definition that allows you to conclude that
$\overline{AO} \perp \overline{BO}$ and $\overline{AO} \perp \overline{CO}.$

Use the figure and the given information below to tell what method (SSS, SAS, ASA, AAS, or HL) can be used to prove $\triangle ABO \cong \triangle ACO.$ You need not write the proofs.

10. Given: $\overline{AO} \perp$ plane $M;\ \overline{BO} \cong \overline{CO}$

11. Given: $\overline{AO} \perp$ plane $M;\ \angle B \cong \angle C$

12. Given: $\overline{AO} \perp$ plane $M;\ \overline{AB} \cong \overline{AC}$

B   13. Given: The figure above with no information except that $\overline{AB} \cong \overline{AC}$ and $\overline{OB} \cong \overline{OC}.$
a. Is it possible to prove that $\angle AOB \cong \angle AOC?$
b. Is it possible to prove that $\angle AOB$ and $\angle AOC$ are right angles?

14. Copy the figure for Exercises 9–14 and draw $\overline{BC}.$ Suppose $\overline{AO} \perp \overline{OB}$ and $\overline{AO} \perp \overline{OC}.$ Classify the following as true or false.
a. If $\overline{AB} \cong \overline{AC},$ then $\angle OBC \cong \angle OCB.$
**b.** The converse of the statement in (a).

Tell which pairs of congruent parts and what method (SSS, SAS, ASA, AAS, or HL) you would use to prove the triangles are congruent.

15. Given: $\angle 1 \cong \angle 2;\ \angle 3 \cong \angle 4;\ \angle 5 \cong \angle 6$
$\triangle PQX \cong \triangle PTY$ by what method?

16. Given: $\angle 7 \cong \angle 8;\ \angle 3 \cong \angle 4;\ \overline{QR} \cong \overline{ST}$
$\triangle QPR \cong \triangle TPS$ by what method?
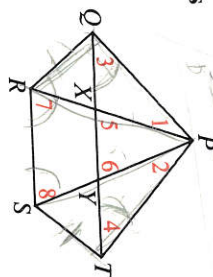
17. a. Draw an isosceles triangle RST with $RS = RT.$ Let M be the midpoint of $\overline{RT}$ and N be the midpoint of $\overline{RS}.$ Draw $\overline{SM}$ and $\overline{TN}$ and label their common point $O.$ Now draw $\overline{MN}.$
b. Name four *pairs* of congruent triangles.

18. Write a two-column proof.
Given: $\angle 1 \cong \angle 2 \cong \angle 3;$
$\qquad \overline{ES} \cong \overline{DT}$

Prove: $\angle 4 \cong \angle 5$
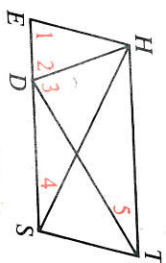
19. Draw an isosceles triangle ABC with $AB = AC.$ Also draw the bisector of $\angle B,$ intersecting $\overline{AC}$ at $X,$ and the bisector of $\angle C,$ intersecting $\overline{AB}$ at $Y.$
Prove that $BX = CY.$

For Exercises 20 and 21, write a detailed plan for proof instead of a two-column proof.

20. Draw an isosceles triangle. From the midpoint of each leg draw a perpendicular segment to the base. Prove that these segments are congruent. (First label your figure and state what is given and what is to be proved.)

21. Given: $\overline{FL} \cong \overline{AK};$
$\qquad \overline{SF} \cong \overline{SK};$
$\qquad M$ is the midpoint of $\overline{SF};$
$\qquad N$ is the midpoint of $\overline{SK}.$

Prove: $\overline{AM} \cong \overline{LN}$
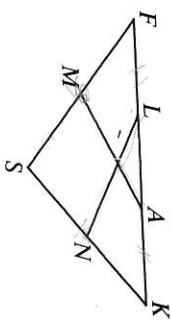
Write proofs in two-column form.

C   22. The diagram shows three squares and an equilateral triangle.
Prove: $AE = FC = ND$

23. Use the results of Exercise 22 to prove that $\triangle FAN$ is equilateral.

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