

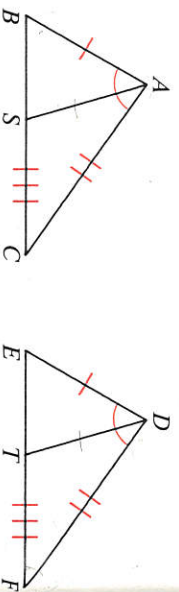
Classroom Exercises

In Exercises 1-3 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.

1. Prove: $\overline{AS} \cong \overline{DT}$

Key steps of proof:

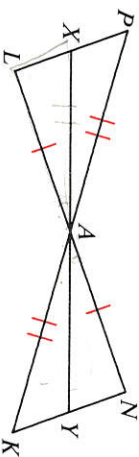
- $\triangle ABC \cong \triangle DEF$
- $\angle C \cong \angle F$
- $\triangle ACS \cong \triangle DFT$
- $\overline{AS} \cong \overline{DT}$



2. Prove: $\overline{AX} \cong \overline{AY}$

Key steps of proof:

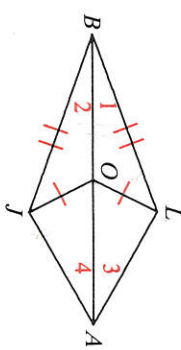
- $\triangle PAL \cong \triangle KAN$
- $\angle L \cong \angle N$
- $\triangle LAX \cong \triangle NAY$
- $\overline{AX} \cong \overline{AY}$



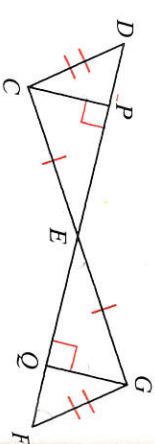
3. Prove: $\angle 3 \cong \angle 4$

Key steps of proof:

- $\triangle LOB \cong \triangle JOB$
- $\angle 1 \cong \angle 2$
- $\triangle LBA \cong \triangle JBA$
- $\angle 3 \cong \angle 4$



4. State a plan for proving that $\angle D \cong \angle F$.



Written Exercises

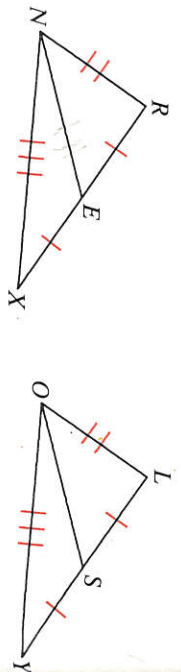
In Exercises 1-5 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.

A

1. Prove: $\overline{NE} \cong \overline{OS}$

Key steps of proof:

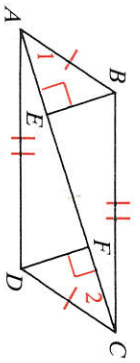
- $\triangle RNX \cong \triangle LOY$
- $\angle X \cong \angle Y$
- $\triangle NEX \cong \triangle OSY$
- $\overline{NE} \cong \overline{OS}$



2. Prove: $\overline{BE} \cong \overline{DF}$

Key steps of proof:

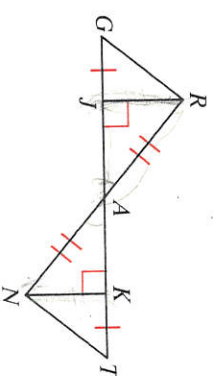
- $\triangle ABC \cong \triangle CDA$
- $\angle 1 \cong \angle 2$
- $\triangle ABE \cong \triangle CDF$
- $\overline{BE} \cong \overline{DF}$



3. Prove: $\angle G \cong \angle T$

Key steps of proof:

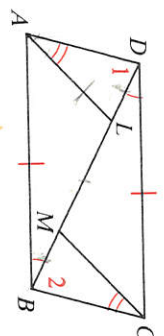
- $\triangle RAL \cong \triangle NAK$
- $\overline{RJ} \cong \overline{NK}$
- $\triangle GRJ \cong \triangle TNK$
- $\angle G \cong \angle T$



4. Prove: $\overline{AL} \cong \overline{CM}$

Key steps of proof:

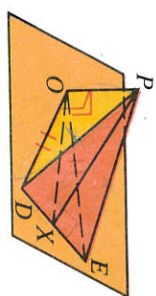
- $\triangle ABD \cong \triangle CDB$
- $\overline{AD} \cong \overline{CB}$; $\angle 1 \cong \angle 2$
- $\triangle ADL \cong \triangle CBM$
- $\overline{AL} \cong \overline{CM}$



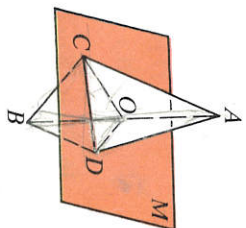
5. Prove: $\overline{DX} \cong \overline{EX}$

Key steps of proof:

- $\triangle POD \cong \triangle POE$
- $\overline{PD} \cong \overline{PE}$
- $\triangle PDX \cong \triangle PEX$
- $\overline{DX} \cong \overline{EX}$



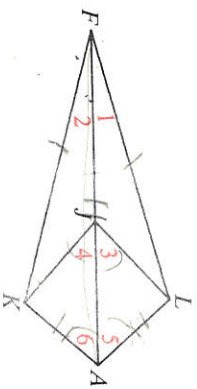
6. Plane M is the perpendicular bisecting plane of \overline{AB} at O (that is, the plane that is perpendicular to \overline{AB} at its midpoint, O). Points C and D also lie in plane M . List three pairs of congruent triangles and tell which congruence method can be used to prove each pair congruent.



B

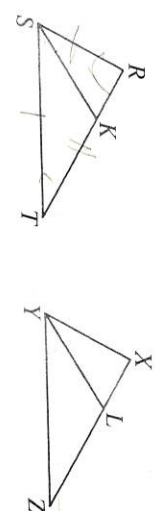
7. Given: $\overline{FL} \cong \overline{FK}$; $\overline{LA} \cong \overline{KA}$

Prove: $\overline{LJ} \cong \overline{KJ}$



8. Given: \overline{FA} bisects $\angle LFK$ and $\angle LAK$.

Prove: \overline{FA} bisects $\angle LJK$.

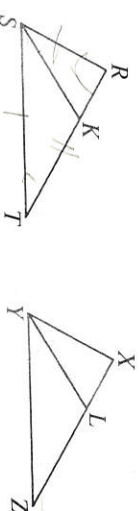


9. Given: $\triangle RST \cong \triangle XYZ$;

\overline{SK} bisects $\angle RST$;

\overline{YL} bisects $\angle XYZ$.

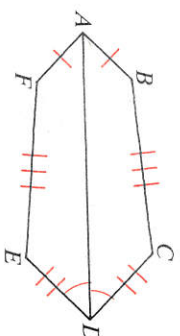
Prove: $\overline{SK} \cong \overline{YL}$



10. Given: Congruent parts as marked in the diagram.

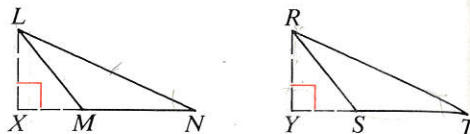
Prove: $\angle B \cong \angle F$

(Hint: First draw two auxiliary lines.)

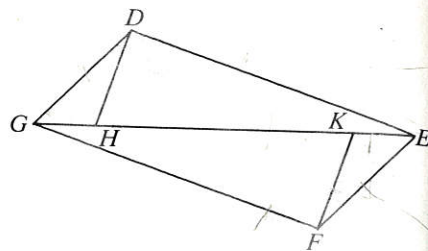


11. Draw two congruent acute triangles, $\triangle PAY$ and $\triangle NOW$. Draw the altitudes \overline{PE} and \overline{NF} and prove that they are congruent. (First state what is given and what is to be proved.)

12. Given: $\triangle LMN \cong \triangle RST$;
 \overline{LX} and \overline{RY} are altitudes.
 Prove: $\overline{LX} \cong \overline{RY}$



13. Given: $\overline{DE} \cong \overline{FG}$; $\overline{GD} \cong \overline{EF}$;
 $\angle HDE$ and $\angle KFG$ are right angles.
 Prove: $\overline{DH} \cong \overline{FK}$



14. Given: $\overline{GD} \parallel \overline{EF}$; $\angle GDE \cong \angle GFE$;
 $\overline{GH} \cong \overline{EK}$
 Prove: $\overline{DH} \parallel \overline{FK}$

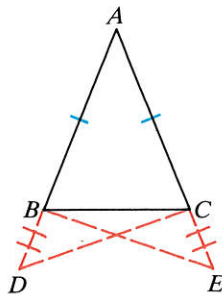
15. Draw two line segments, \overline{SX} and \overline{TY} , that bisect each other at O . Choose any point P on \overline{ST} and let Q be the point where \overline{PO} intersects \overline{XY} . Prove that O is the midpoint of \overline{PQ} . (First state what is given and what is to be proved.)

16. This figure is like the one that Euclid used to prove that the base angles of an isosceles triangle are congruent (our Theorem 3-1). Write a paragraph proof following his key steps shown below.

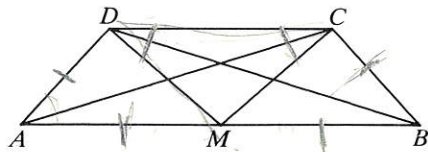
- Given: $\overline{AB} \cong \overline{AC}$;
 \overline{AB} and \overline{AC} are extended so $\overline{BD} \cong \overline{CE}$.
 Prove: $\angle ABC \cong \angle ACB$

Euclid's Key Steps

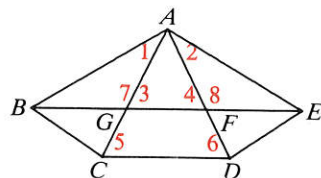
1. Prove $\triangle DAC \cong \triangle EAB$.
2. Prove $\triangle DBC \cong \triangle ECB$.
3. Prove $\angle DBC \cong \angle ECB$ and then $\angle ABC \cong \angle ACB$.



- C 17. Given: $\overline{AM} \cong \overline{MB}$; $\overline{AD} \cong \overline{BC}$;
 $\angle MDC \cong \angle MCD$
 Prove: $\overline{AC} \cong \overline{BD}$



18. Given: $\angle 1 \cong \angle 2$;
 $\angle 3 \cong \angle 4$;
 $\angle 5 \cong \angle 6$
 Prove: $\overline{BC} \cong \overline{ED}$

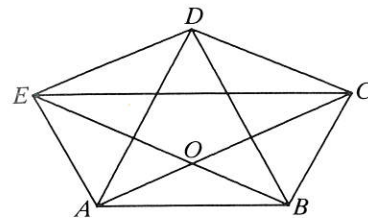


19. A, B, C , and D are noncoplanar. $\triangle ABC$, $\triangle BCD$, $\triangle CAD$, and $\triangle ABD$ are equilateral. X and Y are midpoints of \overline{AC} and \overline{BD} respectively. Z is the midpoint of \overline{XY} . What kind of triangle is $\triangle XYZ$?

20. Given: \overline{SN} and \overline{TM} are medians of scalene $\triangle PQR$.
 P is on \overline{SN} such that $\overline{SN} \cong \overline{NP}$;
 Q is on \overline{TM} such that $\overline{TM} \cong \overline{MQ}$.
 Prove: a. $\overline{RQ} \cong \overline{RP}$
 b. \overline{RQ} and \overline{RP} are both parallel to \overline{PQ} .
 c. P, R , and Q are collinear.

For Exercises 21-23, write paragraph proofs. (Exercise 21 is unusually difficult.)

- ★ 21. Given: $\overline{AE} \parallel \overline{BD}$; $\overline{BC} \parallel \overline{AD}$;
 $\overline{AE} \cong \overline{BC}$; $\overline{AD} \cong \overline{BD}$
 Prove: a. $\overline{AC} \cong \overline{BE}$
 b. $\overline{EC} \parallel \overline{AB}$



- ★ 23. Given: X, R, S , and T are coplanar;
 X is the midpoint of \overline{AB} ;
 $\overline{AB} \perp \overline{RX}$; $\overline{AB} \perp \overline{TX}$
 Prove: $\overline{AB} \perp \overline{SX}$

Note: This exercise proves that if a line (\overline{AB}) is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to every line of the plane through that point. Thus the line is perpendicular to the plane.

3-8 Inductive Reasoning

Throughout these first three chapters, we have been using **deductive reasoning**. Now we'll consider **inductive reasoning**, a kind of reasoning used in science and in everyday life.

Example 1 After picking marigolds for the garden, a girl sneezed. She also began sneezing when she was near marigolds. Based on this, she concluded inductively that she is allergic to marigolds.