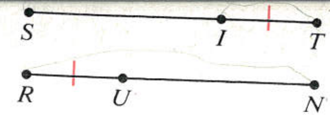
Prove: $SI = UN$



Proof:

Statements	Reasons
1. $ST = RN$	1. ?
2. ? $= SI + IT$; ? $= RU + UN$	2. ?
3. $SI + IT = RU + UN$	3. ?
4. $IT = RU$	4. ?
5. ?	5. ?

Written Exercises

Justify each step.

A

1. $4x - 5 = -2$
$4x = 3$
$x = \frac{3}{4}$

2. $\frac{3a}{2} = \frac{6}{5}$
$3a = \frac{12}{5}$
$a = \frac{4}{5}$

3. $\frac{z+7}{3} = -11$
$z + 7 = -33$
$z = -40$

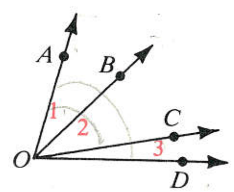
4. $15y + 7 = 12 - 20y$
$35y + 7 = 12$
$35y = 5$
$y = \frac{1}{7}$

5. $\frac{2}{3}b = 8 - 2b$
$2b = 3(8 - 2b)$
$2b = 24 - 6b$
$8b = 24$
$b = 3$

6. $\frac{x-2}{2} = \frac{4+x}{5}$
$5(x - 2) = 2(4 + x)$
$5x - 10 = 8 + 2x$
$3x - 10 = 8$
$3x = 18$
$x = 6$

Copy everything shown and supply any missing statements and reasons.

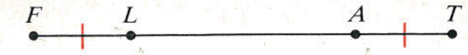
7. Given: $\angle AOD$ as shown
Prove: $m\angle AOD = m\angle 1 + m\angle 2 + m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOD = m\angle AOC + m\angle 3$	1. ?
2. $m\angle AOC = m\angle 1 + m\angle 2$	2. ?
3. ?	3. ?

8. Given: $FL = AT$
Prove: $FA = LT$



Proof:

Statements	Reasons
1. ?	1. Given
2. $LA = LA$	2. ?
3. $FL + LA = AT + LA$	3. ?
4. $FL + LA = FA$; $LA + AT = LT$	4. ?
5. ?	5. Substitution Prop.

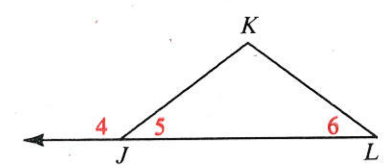
9. Given: $DW = ON$
Prove: $DO = WN$

Proof:

Statements	Reasons
1. $DW = ON$	1. ?
2. $DW = DO + OW$; $ON =$? + ?	2. ?
3. ?	3. Substitution Prop.
4. $OW = OW$	4. ?
5. ?	5. ?

B

10. Given: $m\angle 4 + m\angle 6 = 180$
Prove: $m\angle 5 = m\angle 6$



Proof:

Statements	Reasons
1. $m\angle 4 + m\angle 6 = 180$	1. ?
2. $m\angle 4 + m\angle 5 = 180$	2. ?
3. $m\angle 4 + m\angle 5 = m\angle 4 + m\angle 6$	3. ?
4. $m\angle 4 = m\angle 4$	4. ?
5. ?	5. ?

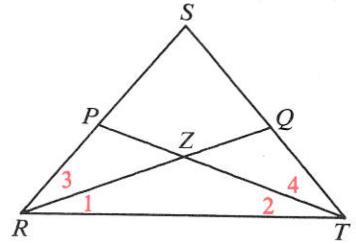
Copy everything shown and write a two-column proof.

11. Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$
Prove: $m\angle SRT = m\angle STR$

12. Given: $RP = TQ$; $PS = QS$
Prove: $RS = TS$

13. Given: $RQ = TP$; $ZQ = ZP$
Prove: $RZ = TZ$

14. Given: $m\angle SRT = m\angle STR$; $m\angle 3 = m\angle 4$
Prove: $m\angle 1 = m\angle 2$



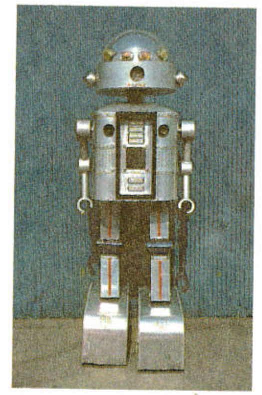
Exs. 11–14

C **15.** Consider the following statements:

Reflexive Property: Robot A is as rusty as itself.
Symmetric Property: If Robot A is as rusty as Robot B, then Robot B is as rusty as Robot A.
Transitive Property: If Robot A is as rusty as Robot B and Robot B is as rusty as Robot C, then Robot A is as rusty as Robot C.

A *relation* that is reflexive, symmetric, and transitive is an *equivalence relation.* The relation "is as rusty as" is an equivalence relation. Which of the following are equivalence relations?

a. is rustier than
b. has the same length as
c. is opposite (for rays)
d. is coplanar with (for lines)



B I O G R A P H I C A L   N O T E

Julia Morgan

Julia Morgan (1872–1959), the first successful woman architect in the United States, was born in San Francisco. Though best known for her design of San Simeon, the castle-like home of William Randolph Hearst that is now the property of the State of California, she designed numerous public buildings and private homes. Even today, to own "a Julia Morgan house" carries considerable prestige.

To become an architect, Morgan needed great determination as well as a brilliant mind. Since the University of California did not have an architecture curriculum at that time, she prepared for graduate work in Paris by studying civil engineering. In Paris the École des Beaux-Arts, which had just begun to admit foreigners, was particularly reluctant to admit a foreign woman. She persisted, however, and became the school's first woman graduate.

1-5 Proving Theorems

Recall that statements that are accepted without proof are called *postulates.* You have already seen statements of the following postulates:

The Ruler Postulate
The Protractor Postulate
The Segment Addition Postulate
The Angle Addition Postulate

We will also accept properties from algebra as postulates in our study of geometry.

Statements that are proved are called *theorems.* Our first theorem uses the definition of a midpoint to prove additional properties of a midpoint that are not explicitly given in the definition. Although this theorem states something very obvious, later theorems in this book will not be so obvious. In fact you may find many of them surprising or amazing.

Theorem 1-1   Midpoint Theorem

If M is the midpoint of $\overline{AB}$, then:

$$2AM = AB \text{ and } AM = \tfrac{1}{2}AB$$
$$2MB = AB \text{ and } MB = \tfrac{1}{2}AB$$

Given: M is the midpoint of $\overline{AB}$.

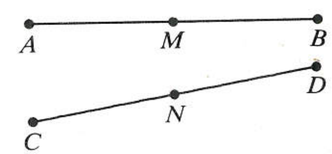
Prove: $2AM = AB$; $AM = \frac{1}{2}AB$;
$2MB = AB$; $MB = \frac{1}{2}AB$

Proof:

Statements	Reasons
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $AM = MB$	2. Definition of midpoint
3. $AM + MB = AB$	3. Segment Addition Postulate
4. $AM + AM = AB$, or $2AM = AB$	4. Substitution Prop.
5. $AM = \frac{1}{2}AB$	5. Division Prop. of =
6. $MB = \frac{1}{2}AB$; $2MB = AB$	6. Substitution Prop. (Steps, 2, 4, and 5)

Example 1 Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{CD}$;
$AB = CD$

What can you conclude?



Solution From the definition of midpoint, you know that $AM = MB$ and $CN = ND$. From the Midpoint Theorem, you know that $AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$. Since $AB = CD$, you can deduce that $\frac{1}{2}AB = \frac{1}{2}CD$. Thus, $AM = MB = CN = ND$.