



TUTANKHAMUN

$a:b$ . This form can also be used to compare three numbers

angle are in the ratio 2:2:5. Find the

es.

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e equal. For example,

m can be read "a is to b" as the proportion. The terms, respectively. an extended propor-

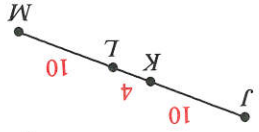
### Classroom Exercises

- Express the ratio in simplest form.
- $\frac{15}{20}$
  - $\frac{7j}{4j}$
  - $\frac{3}{4n}$
  - $\frac{4}{4n^2}$

5. Compare your answers to Exercises 3 and 4. Is the ratio  $a:b$  of two numbers always, sometimes, or never the same as the ratio  $b:a$ ?

Express the ratio in simplest form.

- $JK:KL$
- $KL:JK$
- $KM:LK$
- $JL:LM$
- $JK:KL:LM$
- $KL:JM$
- $JK:KL:LM$



12. What is the ratio of 750 milliliters to 1.5 liters?

13. Can you find the ratio of 2 liters to 4 kilometers? Explain.

14. The ratio of the lengths of two segments is 4:3 when they are measured in centimeters. What is their ratio when they are measured in inches?

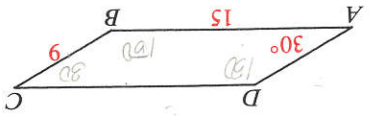
15. Three numbers aren't known, but the ratio of the numbers is 1:2:5. Is it possible that the numbers are 1, 2, and 5? 10, 20, and 50? 3, 6, and 20?  $x$ ,  $2x$ , and  $5x$ ?

16. What is the second term of the proportion  $\frac{a}{b} = \frac{x}{y}$ ?

### Written Exercises

$ABCD$  is a parallelogram. Find the value of each ratio.

- $AB:BC$
- $AB:CD$
- $m\angle C:m\angle D$
- $m\angle B:m\angle C$
- $AD$ : perimeter of  $ABCD$

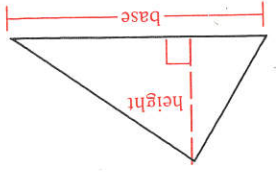


In Exercises 6-14,  $x = 12$ ,  $y = 8$ , and  $z = 24$ . Write each ratio in simplest form.

- $x$  to  $y$
- $z$  to  $x$
- $\frac{z+x}{z-x}$
- $\frac{x}{y}$
- $\frac{x+z}{y}$
- $x:y:z$
- $x:(x+y):(y+z)$
- $x+y$  to  $z$
- $\frac{x-y}{x+y}$
- $x:(x+y):(y+z)$

Exercises 15-20 refer to a triangle. Express the ratio of the height to the base in simplest form.

height	base
1 m	0.8 m
0.4 km	0.3 km
40 cm	2 m
2 cm	5 mm
3 km	150 m
80 mm	0.5 m





Write the algebraic ratio in simplest form.

21.  $\frac{3a}{4ab}$

22.  $\frac{2cd}{5c^2}$

23.  $\frac{3(x+4)}{a(x+4)}$

24.  $\frac{10x}{5x}$

25.  $\frac{3(x-y)}{(x-y)(x+y)}$

26.  $\frac{a+5}{4a+20}$

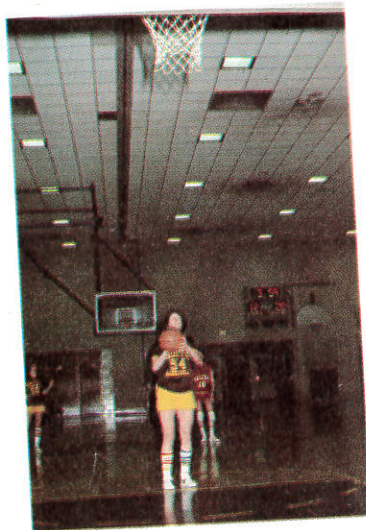
In Exercises 27–32, find the measure of each angle.

- B** 27. The ratio of the measures of two complementary angles is 4:5. (Hint: Let  $4x$  and  $5x$  represent the measures.)
28. The ratio of the measures of two supplementary angles is 11:4.
29. The measures of the angles of a triangle are in the ratio 3:4:5.
30. The measures of the acute angles of a right triangle are in the ratio 5:7.
31. The measures of the angles of an isosceles triangle are in the ratio 3:3:4.
32. The measures of the angles of a hexagon are in the ratio 4:6:6:7:8:9.
33. The perimeter of a triangle is 96 cm and the lengths of its sides are in the ratio 9:11:12. Find the length of each side.
34. The measures of the consecutive angles of a quadrilateral are in the ratio 6:7:11:12. Find the measure of each angle, draw a quadrilateral that satisfies the requirements, and explain why two sides must be parallel.
35. What is the ratio of the measure of an interior angle to the measure of an exterior angle in a regular decagon? A regular  $n$ -gon?
36. A team's best hitter has a lifetime batting average of .320. He has been at bat 325 times.
- How many hits has he made?
  - The player goes into a slump and doesn't get any hits at all in his next ten times at bat. Now what is his batting average to the nearest thousandth?

- C** 37. A basketball player has made 24 points out of 30 free throws. She hopes to make all her next free throws until her free-throw percentage is 85 or better. How many consecutive free throws will she have to make?

38. Points  $B$  and  $C$  lie on  $\overline{AD}$ .  $\frac{AB}{BD} = \frac{3}{4}$ ,  $\frac{AC}{CD} = \frac{5}{6}$ , and  $BD = 66$ . Find  $AC$ .

39. Find the ratio of  $x$  to  $y$ :  $\frac{4}{y} + \frac{3}{x} = 44$   
 $\frac{12}{y} - \frac{2}{x} = 44$



The first and last terms of a proportion are called the *extremes*. The terms in the middle are called the *means*. In the proportions below, the extremes are in red and the means are in black.

$a:b = c:d$

$6:9 = 2:3$

$\frac{6}{9} = \frac{2}{3}$

Notice that  $6 \cdot 3 = 9 \cdot 2$ . This illustrates a property of all proportions: the *means-extremes* property of proportions:

The product of the extremes equals the product of the means.

$\frac{a}{b} = \frac{c}{d}$  is equivalent to  $ad = bc$ .

The two equations are equivalent because we can change either equation by multiplying (or dividing) each side by  $bd$ . Try it.

It is often necessary to replace one proportion by an equivalent one. When you do so in a proof, you may use the reason: "The following properties will be justified in the next section."

### Properties of Proportions

1.  $\frac{a}{b} = \frac{c}{d}$  is equivalent to:

a.  $ad = bc$       b.  $\frac{a}{c} = \frac{b}{d}$       c.  $\frac{b}{a} = \frac{d}{c}$       d.  $\frac{a}{b} = \frac{c}{d}$

2. If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ , then  $\frac{a+c+e+\dots}{b+d+f+\dots} = \frac{a}{b} = \dots$

**Example** Use the proportion  $\frac{x}{y} = \frac{3}{4}$  to complete each statement.

a.  $3y = 4x$       b.  $\frac{x+y}{y} = \frac{?}{?}$

c.  $\frac{x}{3} = \frac{?}{?}$       d.  $\frac{4}{3} = \frac{?}{?}$

**Solution**

a.  $3y = 4x$       b.  $\frac{x+y}{y} = \frac{7}{4}$

c.  $\frac{x}{3} = \frac{y}{4}$       d.  $\frac{4}{3} = \frac{y}{x}$



1. If  $\frac{f}{e} = \frac{h}{g}$ , which equation is correct?  
 a.  $ef = gh$   
 b.  $eh = fg$

2. Which proportions are equivalent to  $\frac{12}{x} = \frac{4}{3}$ ?  
 a.  $\frac{3}{x} = \frac{4}{12}$   
 b.  $\frac{4}{x} = \frac{12}{3}$

c.  $\frac{12}{x} = \frac{3}{4}$

d.  $\frac{x}{12} + \frac{12}{x} = \frac{4}{7}$

Complete the statement.

3. If  $\frac{b}{a} = \frac{5}{6}$ , then  $5a = \frac{?}{?}$ .

5. If  $\frac{f}{e} = \frac{11}{7}$ , then  $\frac{7}{e} = \frac{?}{?}$ .

7. If  $\frac{b}{a} = \frac{3}{2}$ , then  $\frac{a+b}{b} = \frac{?}{?}$ .

9. a. Apply the means-extremes property of proportions to the proportion  $\frac{f}{e} = \frac{5}{8}$  and you get  $5e = \frac{?}{?}$ .

b. Apply the property to the proportion  $\frac{f}{5} = \frac{e}{8}$  and you get  $\frac{?}{?} = \frac{?}{?}$ .

c. Are the proportions  $\frac{f}{e} = \frac{5}{8}$  and  $\frac{f}{5} = \frac{e}{8}$  equivalent?

10. Explain an easy way to show that the proportions  $\frac{7}{x} = \frac{3}{2}$  and  $\frac{2}{x} = \frac{3}{7}$  are not equivalent.

What can you conclude from the given information?

11.  $\frac{b}{a} = \frac{n}{c}$  and  $\frac{a}{b} = \frac{c}{x}$

12.  $\frac{4}{3} = \frac{k}{y}$  and  $\frac{v}{3} = \frac{k}{4}$

13. Apply the means-extremes property to  $\frac{b}{a} = \frac{d}{c}$  and also to  $\frac{c}{a} = \frac{d}{b}$ . (Note that you have justified Property 1(b) on page 209 by showing that each proportion is equivalent to the same equation.)

14. Explain why  $\frac{b}{a} = \frac{d}{c}$  and  $\frac{a}{b} = \frac{d}{c}$  are equivalent. (This justifies Property 1(c) on page 209.)

Written Exercises

Complete each statement.

1. If  $\frac{5}{x} = \frac{4}{3}$ , then  $4x = \frac{?}{?}$ .

3. If  $n:3 = 7:8$ , then  $8n = \frac{?}{?}$ .

2. If  $\frac{x}{7} = \frac{8}{3}$ , then  $3x = \frac{?}{?}$ .

4. If  $4:g = 5:6$ , then  $5g = \frac{?}{?}$ .

A

1. If  $\frac{f}{e} = \frac{h}{g}$ , which equation is correct?  
 a.  $ef = gh$   
 b.  $eh = fg$

2. Which proportions are equivalent to  $\frac{12}{x} = \frac{4}{3}$ ?  
 a.  $\frac{3}{x} = \frac{4}{12}$   
 b.  $\frac{4}{x} = \frac{12}{3}$

c.  $\frac{12}{x} = \frac{3}{4}$

d.  $\frac{x}{12} + \frac{12}{x} = \frac{4}{7}$

Complete the statement.

3. If  $\frac{b}{a} = \frac{5}{6}$ , then  $5a = \frac{?}{?}$ .

5. If  $\frac{f}{e} = \frac{11}{7}$ , then  $\frac{7}{e} = \frac{?}{?}$ .

7. If  $\frac{b}{a} = \frac{3}{2}$ , then  $\frac{a+b}{b} = \frac{?}{?}$ .

9. a. Apply the means-extremes property of proportions to the proportion  $\frac{f}{e} = \frac{5}{8}$  and you get  $5e = \frac{?}{?}$ .

b. Apply the property to the proportion  $\frac{f}{5} = \frac{e}{8}$  and you get  $\frac{?}{?} = \frac{?}{?}$ .

c. Are the proportions  $\frac{f}{e} = \frac{5}{8}$  and  $\frac{f}{5} = \frac{e}{8}$  equivalent?

10. Explain an easy way to show that the proportions  $\frac{7}{x} = \frac{3}{2}$  and  $\frac{2}{x} = \frac{3}{7}$  are not equivalent.

What can you conclude from the given information?

11.  $\frac{b}{a} = \frac{n}{c}$  and  $\frac{a}{b} = \frac{c}{x}$

12.  $\frac{4}{3} = \frac{k}{y}$  and  $\frac{v}{3} = \frac{k}{4}$

13. Apply the means-extremes property to  $\frac{b}{a} = \frac{d}{c}$  and also to  $\frac{c}{a} = \frac{d}{b}$ . (Note that you have justified Property 1(b) on page 209 by showing that each proportion is equivalent to the same equation.)

14. Explain why  $\frac{b}{a} = \frac{d}{c}$  and  $\frac{a}{b} = \frac{d}{c}$  are equivalent. (This justifies Property 1(c) on page 209.)

6. If  $\frac{y}{x} = \frac{3}{8}$ , then  $\frac{x}{y} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{5-x}{x}$ , then  $\frac{a+b}{b} = \frac{?}{?}$ .

10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$

13.  $\frac{x}{x+5} = \frac{4}{1} = \frac{2}{?}$

14.  $\frac{x}{x+3} = \frac{2}{3} = \frac{3}{?}$

17.  $\frac{x}{x+3} = \frac{2}{2x-1} = \frac{3}{?}$

20.  $\frac{4x-5}{4x-5} = \frac{4}{20x+1} = \frac{?}{7}$

19.  $\frac{x}{x+5} = \frac{4}{7}$

16.  $\frac{2x+1}{2x+1} = \frac{3}{2} = \frac{4x-1}{?}$

15.  $\frac{x+3}{x+2} = \frac{4}{?}$

18.  $\frac{7x+5}{10} = \frac{6x-2}{7}$

9.  $\frac{3}{x} = \frac{5}{4}$

12.  $\frac{x}{8} = \frac{5}{2}$

4. If  $\frac{d}{c} = \frac{4}{9}$ , then  $\frac{c}{d} = \frac{?}{?}$ .

6. If  $\frac{x}{w} = \frac{z}{r}$ , then  $\frac{y}{w} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{j}{i}$ , then  $\frac{c}{a} = \frac{?}{?}$ .

15.  $\frac{x+2}{x+2} = \frac{4}{4}$

18.  $\frac{7x+5}{10} = \frac{6x-2}{7}$

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15.  $\frac{x+2}{x+2} = \frac{4}{4}$

18.  $\frac{7x+5}{10} = \frac{6x-2}{7}$

5. If  $\frac{a}{b} = \frac{4}{7}$ , then  $\frac{b}{a} = \frac{?}{?}$ .

7. If  $\frac{2}{x} = \frac{3}{y}$ , then  $\frac{x+2}{x+2} = \frac{?}{?}$ .

10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$

13.  $\frac{x}{x+5} = \frac{4}{1} = \frac{2}{?}$

14.  $\frac{x}{x+3} = \frac{2}{3} = \frac{3}{?}$

17.  $\frac{x}{x+3} = \frac{2}{2x-1} = \frac{3}{?}$

20.  $\frac{4x-5}{4x-5} = \frac{4}{20x+1} = \frac{?}{7}$

19.  $\frac{x}{x+5} = \frac{4}{7}$

16.  $\frac{2x+1}{2x+1} = \frac{3}{2} = \frac{4x-1}{?}$

15.  $\frac{x+3}{x+2} = \frac{4}{?}$

9.  $\frac{3}{x} = \frac{5}{4}$

12.  $\frac{x}{8} = \frac{5}{2}$

4. If  $\frac{d}{c} = \frac{4}{9}$ , then  $\frac{c}{d} = \frac{?}{?}$ .

6. If  $\frac{x}{w} = \frac{z}{r}$ , then  $\frac{y}{w} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{j}{i}$ , then  $\frac{c}{a} = \frac{?}{?}$ .

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15.  $\frac{x+2}{x+2} = \frac{4}{4}$

5. If  $\frac{a}{b} = \frac{4}{7}$ , then  $\frac{b}{a} = \frac{?}{?}$ .

7. If  $\frac{2}{x} = \frac{3}{y}$ , then  $\frac{x+2}{x+2} = \frac{?}{?}$ .

10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$

13.  $\frac{x}{x+5} = \frac{4}{1} = \frac{2}{?}$

14.  $\frac{x}{x+3} = \frac{2}{3} = \frac{3}{?}$

17.  $\frac{x}{x+3} = \frac{2}{2x-1} = \frac{3}{?}$

20.  $\frac{4x-5}{4x-5} = \frac{4}{20x+1} = \frac{?}{7}$

19.  $\frac{x}{x+5} = \frac{4}{7}$

16.  $\frac{2x+1}{2x+1} = \frac{3}{2} = \frac{4x-1}{?}$

15.  $\frac{x+3}{x+2} = \frac{4}{?}$

9.  $\frac{3}{x} = \frac{5}{4}$

12.  $\frac{x}{8} = \frac{5}{2}$

4. If  $\frac{d}{c} = \frac{4}{9}$ , then  $\frac{c}{d} = \frac{?}{?}$ .

6. If  $\frac{x}{w} = \frac{z}{r}$ , then  $\frac{y}{w} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{j}{i}$ , then  $\frac{c}{a} = \frac{?}{?}$ .

15.  $\frac{x+2}{x+2} = \frac{4}{4}$

18.  $\frac{7x+5}{10} = \frac{6x-2}{7}$

9.  $\frac{3}{x} = \frac{5}{4}$

12.  $\frac{x}{8} = \frac{5}{2}$

4. If  $\frac{d}{c} = \frac{4}{9}$ , then  $\frac{c}{d} = \frac{?}{?}$ .

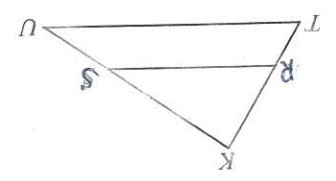
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15.  $\frac{x+2}{x+2} = \frac{4}{4}$

B

21.	12	9	?	16	?	?
22.	8	?	?	10	12	?
23.	16	?	?	?	10	30
24.	?	?	2	?	?	12
25.	?	?	?	12	10	?
26.	12	4	?	?	?	20
27.	?	9	?	36	?	48
28.	?	?	?	30	28	42



(Hint for Ex. 25: Let  $KR = x$ ; then  $RT = 12 - x$ .)

For the figure shown, it is given that  $\frac{KR}{RT} = \frac{KS}{SU}$ . Copy and complete the table.

- 10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$
- 13.  $\frac{x}{x+5} = \frac{4}{1} = \frac{2}{?}$
- 14.  $\frac{x}{x+3} = \frac{2}{3} = \frac{3}{?}$
- 17.  $\frac{x}{x+3} = \frac{2}{2x-1} = \frac{3}{?}$
- 20.  $\frac{4x-5}{4x-5} = \frac{4}{20x+1} = \frac{?}{7}$

8. If  $\frac{b}{a} = \frac{5-x}{x}$ , then  $\frac{a+b}{b} = \frac{?}{?}$ .

6. If  $\frac{y}{x} = \frac{3}{8}$ , then  $\frac{x}{y} = \frac{?}{?}$ .

5. If  $\frac{a}{b} = \frac{4}{7}$ , then  $\frac{b}{a} = \frac{?}{?}$ .

7. If  $\frac{2}{x} = \frac{3}{y}$ , then  $\frac{x+2}{x+2} = \frac{?}{?}$ .

10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$

13.  $\frac{x}{x+5} = \frac{4}{1} = \frac{2}{?}$

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15.  $\frac{x+3}{x+2} = \frac{4}{?}$

9.  $\frac{3}{x} = \frac{5}{4}$

12.  $\frac{x}{8} = \frac{5}{2}$

4. If  $\frac{d}{c} = \frac{4}{9}$ , then  $\frac{c}{d} = \frac{?}{?}$ .

6. If  $\frac{x}{w} = \frac{z}{r}$ , then  $\frac{y}{w} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{j}{i}$ , then  $\frac{c}{a} = \frac{?}{?}$ .

15.  $\frac{x+2}{x+2} = \frac{4}{4}$

18.  $\frac{7x+5}{10} = \frac{6x-2}{7}$

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5. If  $\frac{a}{b} = \frac{4}{7}$ , then  $\frac{b}{a} = \frac{?}{?}$ .

7. If  $\frac{2}{x} = \frac{3}{y}$ , then  $\frac{x+2}{x+2} = \frac{?}{?}$ .

10.  $\frac{x}{x} = \frac{7}{8} = \frac{3}{?}$

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6. If  $\frac{x}{w} = \frac{z}{r}$ , then  $\frac{y}{w} = \frac{?}{?}$ .

8. If  $\frac{b}{a} = \frac{j}{i}$ , then  $\frac{c}{a} = \frac{?}{?}$ .

15.  $\frac{x+$