13. M is the midpoint of $\overline{PQ}$. Complete the table.

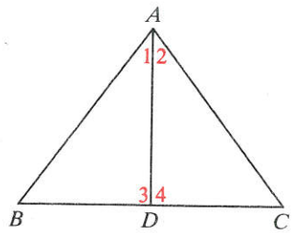
Coordinate of P	1	19	-2	a	1	b
Coordinate of Q	25	7	24	$3a$	?	?
Coordinate of M	?	?	?	?	-2	$4b$

Written Exercises

Name the definition, postulate, or theorem that justifies the statement about the diagram.

A

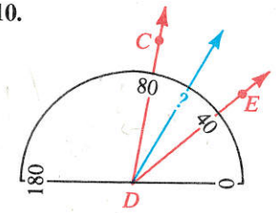
1. If D is the midpoint of $\overline{BC}$, then $\overline{BD} \cong \overline{DC}$.
2. If $\angle 1 \cong \angle 2$, then $\overrightarrow{AD}$ is the bisector of $\angle BAC$.
3. If $\overrightarrow{AD}$ bisects $\angle BAC$, then $m\angle 1 = \frac{1}{2}m\angle BAC$.
4. $m\angle 3 + m\angle 4 = 180$
5. If $BD = DC$, then D is the midpoint of $\overline{BC}$.
6. If D is the midpoint of $\overline{BC}$, then $2BD = BC$.
7. $m\angle 1 + m\angle 2 = m\angle BAC$
8. $BD + DC = BC$
9. If $\overrightarrow{AD}$ is the bisector of $\angle BAC$, then $m\angle 1 = m\angle 2$.



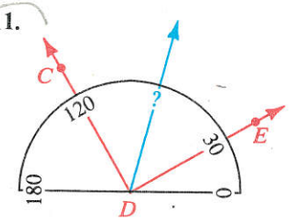
Exs. 1–9

In the diagrams below, what number is paired with the bisector of $\angle CDE$?

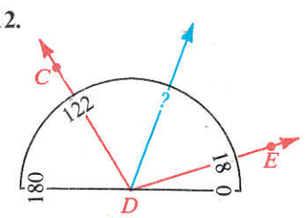
10.



11.



12.



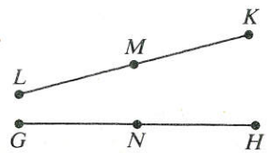
***M* is the midpoint of $\overline{AB}$. Complete.**

	13.	**14.**	**15.**	**16.**	**17.**	**18.**
Coordinate of A	0	24	-12	-3	2	?
Coordinate of B	17	6	-28	13	?	4
Coordinate of M	?	?	?	?	20	-6

B

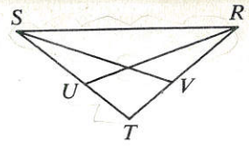
19. a. Suppose M and N are the midpoints of $\overline{LK}$ and $\overline{GH}$, respectively. What segments are congruent?

b. What additional information would enable you to deduce that $LM = NH$?



20. a. Suppose $\overrightarrow{SV}$ bisects $\angle RST$ and $\overrightarrow{RU}$ bisects $\angle SRT$. What angles are congruent?

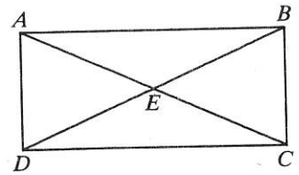
b. What additional information would enable you to deduce that $m\angle VSU = m\angle URV$?



What can you conclude from the given information?

21. Given: $AE = DE$;
$CE = BE$

22. Given: $\overline{AC}$ bisects $\overline{DB}$;
$\overline{DB}$ bisects $\overline{AC}$;
$CE = BE$



Exs. 21, 22

23. Point N is the midpoint of $\overline{LX}$, and point Y is the midpoint of $\overline{LN}$. The coordinates of L and X are 16 and 40, respectively. Sketch a diagram and find:

a. LN **b.** LY

c. The coordinate of Y **d.** The coordinate of N

24. $\overrightarrow{SW}$ is the bisector of $\angle RST$, $\overrightarrow{SZ}$ is the bisector of $\angle RSW$, and $\overrightarrow{SR}$ is the bisector of $\angle NSW$. If $m\angle RST = 72$, find:

a. $m\angle RSZ$ **b.** $m\angle NSZ$

25. Copy and complete the following proof of the statement: If points A and B have coordinates a and b, with $b > a$, and midpoint M of $\overline{AB}$ has coordinate x, then $x = \frac{a+b}{2}$.

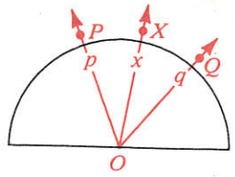
Given: Points A and B have coordinates a and b; $b > a$; midpoint M of $\overline{AB}$ has coordinate x.

Prove: $x = \frac{a+b}{2}$

Proof:

Statements	Reasons
1. A, M, and B have coordinates a, x, and b, respectively; $b > a$	1. ?
2. $AM = x - a$; $MB = b - x$	2. ?
3. M is the midpoint *of* $\overline{AB}$.	3. ?
4. $AM = MB$	4. ?
5. $x - a = b - x$	5. ?
6. $2x =$?	6. ?
7. $x = \frac{a+b}{2}$	7. ?

26. Suppose $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are paired with the real numbers p and q and that $p > q$. Let $\overrightarrow{OX}$, the bisector of $\angle POQ$, be paired with the real number x. Derive an expression for x in terms of p and q.

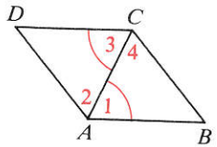


27. $\overrightarrow{BD}$ bisects $\angle ABC$ and $m\angle ABD = x$. Write an equation or inequality that describes x if $\angle ABC$ is:

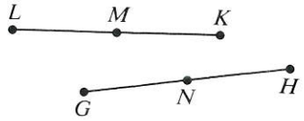
a. a right angle **b.** an acute angle **c.** an obtuse angle

Copy everything shown and write a two-column proof.

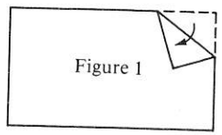
28. Given: $\overrightarrow{AC}$ bisects $\angle DAB$;
$\overrightarrow{CA}$ bisects $\angle BCD$;
$m\angle 1 = m\angle 3$
Prove: $m\angle DAB = m\angle DCB$

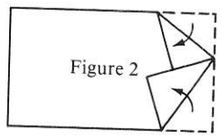


29. Given: M is the midpoint of $\overline{LK}$;
N is the midpoint of $\overline{GH}$;
$LK = GH$
Prove: $MK = NH$



C **30.** Fold down a corner of a rectangular sheet of paper as in Figure 1. Then fold the next corner so that the edges touch as in Figure 2.





Open the paper and measure the angle between the fold lines. Repeat with another sheet of paper, folding the corner at a different angle. Explain why the measured angle is the same in both cases.

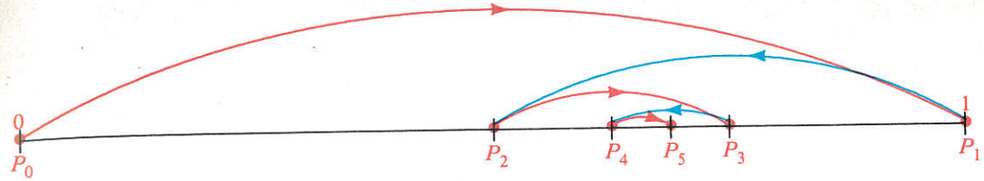
31. Point T is the midpoint of $\overline{RS}$, W is the midpoint of $\overline{RT}$, and Z is the midpoint of $\overline{WS}$. If the length of $\overline{TZ}$ is x, find the following lengths in terms of x.

a. RW **b.** ZS **c.** RS **d.** WZ

(*Hint:* Sketch a diagram and let $y = WT$.)

COMPUTER KEY-IN

A bee starts at point P_0, flies to point P_1, and lands there. The bee then returns half of the way to P_0, landing at P_2. From P_2, the bee returns half of the way to P_1, landing at P_3, and so forth. Can you predict the bee's location after 10 trips?



Assuming that P_0 and P_1 have coordinates 0 and 1, respectively, the BASIC program below will compute and print the bee's location at the end of trips 2 through 10. Notice that P_n represents the position of the bee after n trips. Since P_n is the midpoint of the bee's previous two positions, P_{n-1} and P_{n-2}, line 50 calculates $P(N)$ by using the statement proved in Exercise 25, page 27.

```
10  DIM P(50)
20  LET P(0) = 0
30  LET P(1) = 1
40  FOR N = 2 TO 10 STEP 1
50  LET P(N) = (1/2) * (P(N - 2) + P(N - 1))
60  PRINT N, P(N)
70  NEXT N
80  END
```

Exercises

1. Enter the program on your computer and RUN it. Do you notice any patterns or trends in the coordinates? Change line 40 so that the computer will print the coordinates up to P_{40}. What simple fraction is approximated by P_{40}?

2. In line 50, $P(n)$ could instead be computed from the *series*

$$1 - \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} + \cdots + (-\tfrac{1}{2})^{n-1}$$

where each term of the series reflects the bee's return half of the way from P_{n-1} to P_{n-2}. Thus, line 50 could be replaced by

```
50  P(N) = P(N - 1) + (-1/2) ↑ (N - 1).
```

Change your line 50 and RUN the new program. Check to make sure that both programs produce the same results. (Some slight variations will be expected, due to rounding off.)

3. (Optional) What series, similar to the one in Exercise 2, do you think would reflect the bee's movements if on each trip, it returned one-third of the way to the previous point instead of half of the way? Use this series to modify your program to produce coordinates for the bee's new flight pattern. RUN the program for 30 trips. Determine what point the bee is approaching. What simple fraction is the coordinate of this point?

4. (Optional) Repeat Exercise 3 for a bee that returns one-fourth of the way to the previous point.