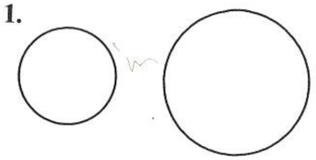
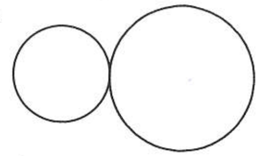
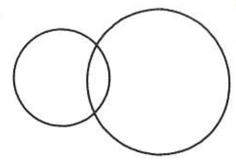
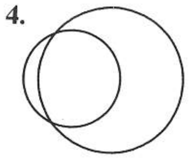
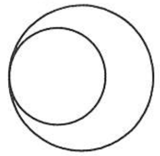
How many common external tangents and how many common internal tangents can be drawn to the two circles?

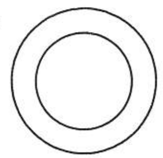
1. 

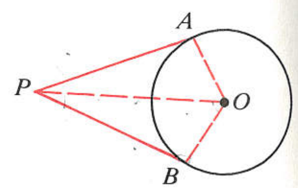
2. 

3. 

4. 

5. 

6. 



7. Write a proof of the corollary to Theorem 7-1.

Given: $\overline{PA}$ and $\overline{PB}$ are tangents to $\odot O$.

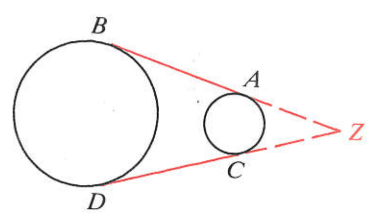
Prove: $\overline{PA} \cong \overline{PB}$

Plan for Proof: Draw $\overline{AO}$, $\overline{BO}$, and $\overline{PO}$. Show that two triangles are congruent and use corresponding parts.

8. State and prove a relation between $\angle APB$ and $\angle AOB$.

Written Exercises

A



1. Copy and complete this proof that the common external tangent segments to two noncongruent circles are congruent.

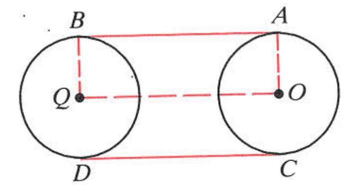
Given: Tangents $\overline{AB}$ and $\overline{CD}$

Prove: $\overline{AB} \cong \overline{CD}$

Proof:

Statements	Reasons
1. Draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, intersecting at Z.	1. Through any two points there is __?__.
2. $ZA + AB = ZB$; $ZC + CD = ZD$	2. __?__
3. $ZB = ZD$	3. __?__
4. $ZA + AB = ZC + CD$	4. __?__
5. $ZA = ZC$	5. __?__
6. $AB = CD$, or $\overline{AB} \cong \overline{CD}$	6. __?__

2. Suppose, in Exercise 1, that the circles are congruent. Then the tangent lines won't meet. Supply reasons for these key steps of the proof that the common tangent segments are congruent for this special case.



Given: Tangents $\overline{AB}$ and $\overline{CD}$;
$\odot O \cong \odot Q$

Prove: $\overline{AB} \cong \overline{CD}$

1. $\overline{OA} \perp \overline{AB}$ and $\overline{QB} \perp \overline{AB}$
2. $\overline{OA} \parallel \overline{QB}$
3. $\overline{OA} \cong \overline{QB}$
4. Quad. $AOQB$ is a $\square$.
5. $\overline{AB} \cong \overline{OQ}$

By a similar proof, $\overline{OQ} \cong \overline{CD}$.

6. $\overline{AB} \cong \overline{CD}$

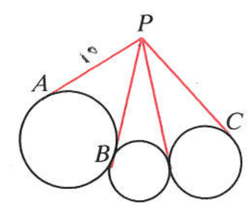
Draw two circles, with all their common tangents, so that the number of common tangents is the stated number.

3. one **4.** two **5.** three **6.** four

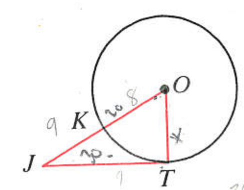
7. How many circles can be tangent to a given line at a given point on the line?

8. Circles O and Q are tangent at point Z. Suppose you draw a different circle tangent to $\odot O$ at Z. What can you say about the new circle and $\odot Q$?

9. The diagram shows tangent circles and lines.
$PA = 10$ $PB =$ __?__ $PC =$ __?__



Ex. 9



Exs. 10–12

In the diagram for Exercises 10–12, $\overline{JT}$ is tangent to $\odot O$.

10. If $JO = 13$ and $OT = 5$, then $JT =$ __?__.

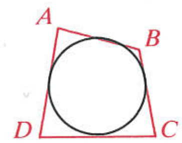
11. If $m\angle OJT = 30$ and $JO = 20$, then $JT =$ __?__.

12. If $JK = 9$ and $KO = 8$, then $JT =$ __?__.

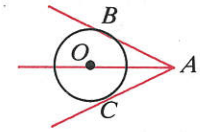
B **13.** Discover and prove a theorem about two lines tangent to a circle at the endpoints of a diameter.

14. State, without proof, a theorem about spheres related to the theorem in Exercise 13.

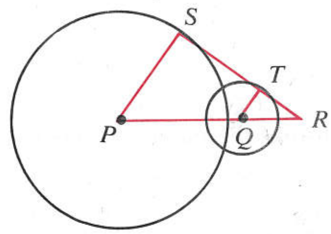
15. Quad. $ABCD$ is circumscribed about a circle. Discover and prove a relationship between $AB + DC$ and $AD + BC$.



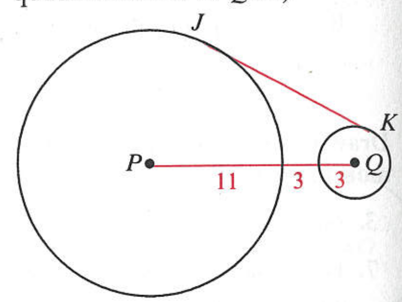
16. Rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangent to $\odot O$. Discover and prove a theorem about $\overrightarrow{AO}$ and $\angle BAC$.



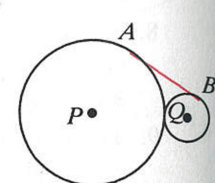
17. $\overline{SR}$ is tangent to $\odot P$ and $\odot Q$.
$QT = 6$; $TR = 8$; $PR = 30$
$PS =$ _?_ $PQ =$ _?_ $ST =$ _?_



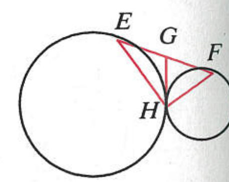
18. $\overline{JK}$ is tangent to $\odot P$ and $\odot Q$.
$JK =$ _?_ (*Hint:* What kind of quadrilateral is $JPQK$?)



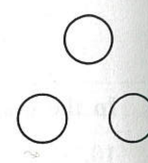
19. Circles P and Q have radii 6 and 2 and are tangent to each other. Find the length of their common external tangent $\overline{AB}$. (*Hint:* Draw $\overline{PQ}$, $\overline{PA}$, and $\overline{QB}$.)



20. Given: Two tangent circles; $\overline{EF}$ is a common external tangent; $\overline{GH}$ is the common internal tangent.
Prove: $\angle EHF$ is a rt. $\angle$.



21. Three circles are shown. How many circles tangent to all three of the given circles can be drawn?



C 22. Suppose the three circles represent three spheres.
   a. How many planes tangent to each of the spheres can be drawn?
   b. How many spheres tangent to each of the three spheres can be drawn?

23. Prove Theorem 7–2. (*Hint:* Write an indirect proof.)

24. Find the radius of the circle inscribed in the triangle.