14. Outline a proof of Case II of Theorem 7-7. Use the diagram on page 312. (*Hint:* Draw the diameter from B and apply Case I.)

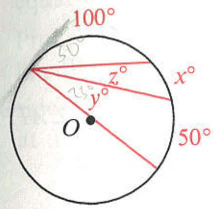
15. Repeat Exercise 14 for Case III.

Written Exercises

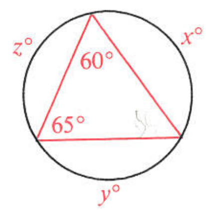
When the letter O is used in a diagram in these exercises, point O is the center of the circle. In Exercises 1-6, find the values of x, y, and z.

A

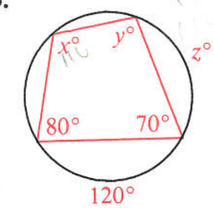
1.



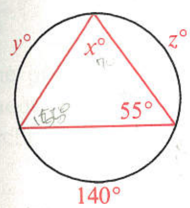
2.



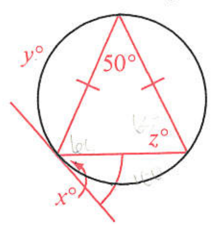
3.



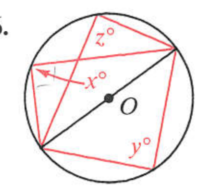
4.



5.



6.

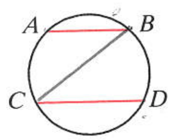


7. Prove: If two chords of a circle are parallel, the two arcs between the chords are congruent.

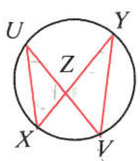
Given: $\overline{AB} \parallel \overline{CD}$

Prove: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$

(*Hint:* Draw $\overline{BC}$.)



8. Prove: $\triangle UXZ \sim \triangle YVZ$

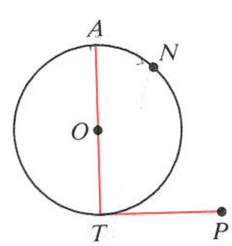


Exercises 9-11 prove the three possible cases of Theorem 7-8. In each case you are given chord $\overline{TA}$ and tangent $\overline{TP}$ of $\odot O$.

9. Supply reasons for the key steps of the proof that $m\angle ATP = \frac{1}{2}m\overset{\frown}{ANT}$ in Case I.

Case I: O lies on $\angle ATP$.

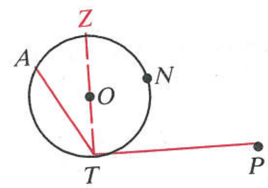
1. $\overline{TP} \perp \overline{TA}$ and $m\angle ATP = 90$.
2. $\overset{\frown}{ANT}$ is a semicircle and $m\overset{\frown}{ANT} = 180$.
3. $\frac{1}{2}m\overset{\frown}{ANT} = 90$
4. $m\angle ATP = \frac{1}{2}m\overset{\frown}{ANT}$



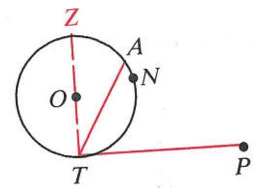
In Case II and Case III, $\overline{AT}$ is not a diameter. You can draw diameter $\overline{TZ}$ and then use Case I, Theorem 7-7, and the Angle Addition and Arc Addition Postulates.

B **10.** Prove $m\angle ATP = \frac{1}{2}m\widehat{ANT}$ in Case II.

11. Prove $m\angle ATP = \frac{1}{2}m\widehat{ANT}$ in Case III.



Case II. O lies inside $\angle ATP$.



Case III. O lies outside $\angle ATP$.

12. Points A, B, C, D, and E are five consecutive vertices of a regular inscribed 15-gon. Chord $\overline{BE}$ is drawn. $m\angle ABE =$ __?__

In Exercises 13 and 14, quadrilateral *FGHJ* is inscribed in a circle. Give numerical answers.

13. $m\angle F = x$, $m\angle G = x$, and $m\angle H = x + 20$. $m\angle J =$ __?__

14. $m\angle F = x^2$, $m\angle G = 9x - 2$, $m\angle H = 11x$, and $m\angle J = x^2 + 20$. The measure of the largest angle of the quadrilateral is __?__.

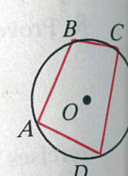
15. The diagram at the right shows a regular polygon with 7 sides.

a. Explain why the numbered angles are all congruent. (*Hint:* You may assume that a circle can be circumscribed about any regular polygon.)

b. Will your reasoning apply to a regular polygon with any number of sides?

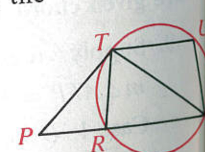
C **16.** Given: Vertices A, B, and C of quadrilateral $ABCD$ lie on $\odot O$; $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$.

Prove: D lies on $\odot O$.



(*Hint:* Use an indirect proof. Assume temporarily that D is not on $\odot O$. You must then treat two cases: (1) D is inside $\odot O$, and (2) D is outside $\odot O$. In each case let X be the point where $\overrightarrow{AD}$ intersects $\odot O$ and draw $\overline{CX}$. Show that what you can conclude about $\angle AXC$ contradicts the given information.)

17. Given: $\overline{PT}$ is a tangent; $\overline{TU} \parallel \overline{PS}$.
Find three similar triangles and prove them similar. Write a paragraph proof.



18. Circle I is inscribed in $\triangle FGH$ and $\odot O$ is circumscribed about $\triangle FGH$. $\overrightarrow{FI}$ intersects $\odot O$ in a point K. Discover and prove a relationship between $\overline{KG}$, $\overline{KH}$, and $\overline{KI}$.