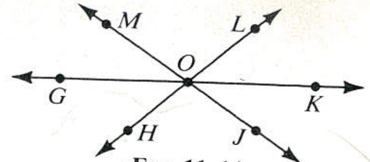
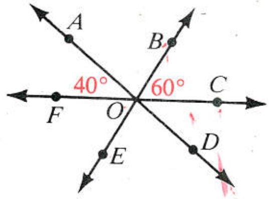
Complete.

11. $\angle GOH \cong$ _?_ 12. $\angle GOM \cong$ _?_
13. $\angle MOK \cong$ _?_ 14. $\angle LOG \cong$ _?_



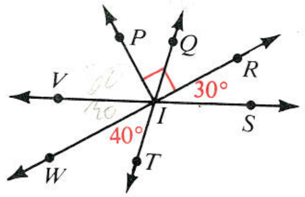
Exs. 11–14

15.



a. $m\angle FOE =$ _?_
b. $m\angle COD =$ _?_
c. $m\angle BOD =$ _?_
d. $m\angle AOB =$ _?_
e. $m\angle DOE =$ _?_

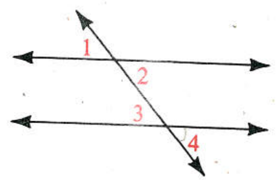
16.



a. $m\angle QIR =$ _?_
b. $m\angle PIQ =$ _?_
c. $m\angle VIT =$ _?_
d. $m\angle VIQ =$ _?_
e. $m\angle SIT =$ _?_

17. A supplement of
   a. an acute angle is _?_. b. an obtuse angle is _?_. c. a right angle is _?_.
18. a. A complement of an acute angle is _?_.
   b. Can a right or an obtuse angle have a complement?

19. Given: $\angle 2 \cong \angle 3$
   a. What can you conclude?
   b. Explain how you would prove your conclusion.



Written Exercises

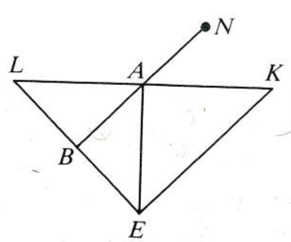
Find the measures of a complement and a supplement of $\angle B$.

A 1. $m\angle B = 55$ 2. $m\angle B = 1$ 3. $m\angle B = 72.5$ 4. $m\angle B = 3x$

5. Two angles are both congruent and complementary. Find their measures.
6. Two angles are both congruent and supplementary. Find their measures.

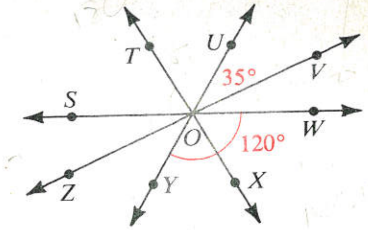
In the diagram, $m\angle KAE = 90$.

7. Name another right angle.
8. Name two congruent supplementary angles.
9. Name two noncongruent supplementary angles.
10. Name two supplementary angles that may or may not be congruent.
11. Name two complementary angles.
12. Name a pair of vertical angles.



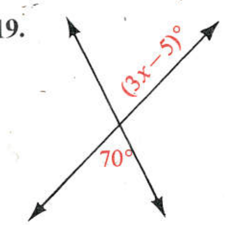
In the diagram, $\overrightarrow{OT}$ bisects $\angle SOU$, $m\angle UOV = 35$, and $m\angle YOW = 120$. Find the measure of each angle.

13. $m\angle ZOY$ 14. $m\angle ZOW$
15. $m\angle VOW$ 16. $m\angle SOU$
17. $m\angle TOU$ 18. $m\angle ZOT$



Find the value of x.

19.



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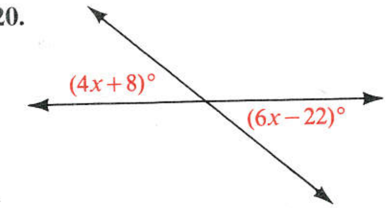
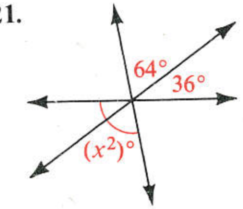


21.



22. $\angle 1$ and $\angle 2$ are supplements.
   $\angle 3$ and $\angle 4$ are supplements.
   a. If $m\angle 1 = m\angle 3 = 27$, find the measures of $\angle 2$ and $\angle 4$.
   b. If $m\angle 1 = m\angle 3 = x$, find the measures of $\angle 2$ and $\angle 4$ in terms of x.
   c. If two angles are congruent, must their supplements be congruent?

If $\angle A$ and $\angle B$ are supplementary, find the value of x and the measures of the angles.

B 23. $m\angle A = 2x$, $m\angle B = x - 15$ 24. $m\angle A = x + 16$, $m\angle B = 2x - 16$

If $\angle C$ and $\angle D$ are complementary, find the value of y and the measures of the angles.

25. $m\angle C = 3y + 5$, $m\angle D = 2y$ 26. $m\angle C = y - 8$, $m\angle D = 3y + 2$

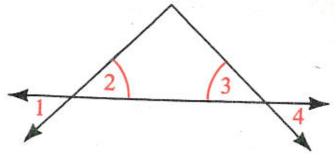
Use the given information to write an equation. Solve the equation to find the measures of the two angles described.

27. A supplement of an angle is twice as large as the angle.
28. A complement of an angle is five times as large as the angle.
29. The measure of one of two complementary angles is six less than twice the measure of the other.
30. The difference between the measures of two supplementary angles is 42.

Find the measures of the angle, its complement, and its supplement.

31. A supplement of an angle is six times as large as a complement of the angle.
32. Three times the measure of a supplement of an angle is eight times the measure of a complement of the angle.

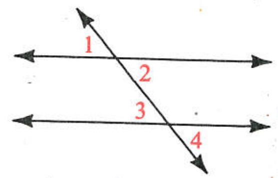
33. Copy everything shown. Complete the proof.
Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



Proof:

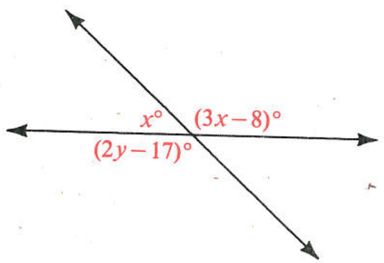
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. _?_
2. $\angle 2 \cong \angle 3$	2. _?_
3. $\angle 3 \cong \angle 4$	3. _?_
4. _?_	4. Transitive Property (used twice)

34. Copy the figure and the statement of what is given and what is to be proved. Then write a two-column proof.
Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



Find the values of *x* and *y* for each diagram.

35.



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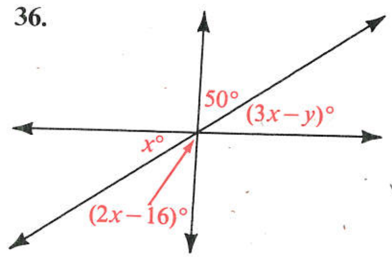
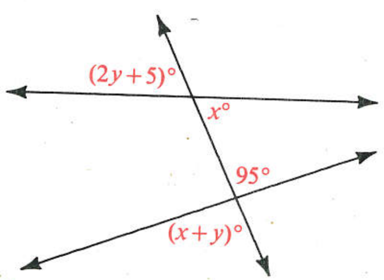
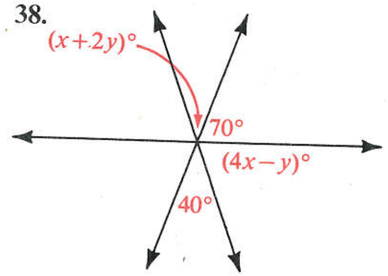


C **37.**



38.



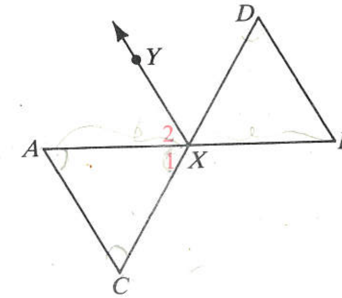
39. Explain why the measure of a complement of an angle can never be exactly half the measure of a supplement of the angle.

40. Describe all angles whose measure is equal to the difference between the measure of a supplement of the angle and twice the measure of a complement of the angle.

Self-Test 2

1. Name the four kinds of reasons that may be used to justify the statements in a proof.

Write the name or the statement of the property or theorem that justifies the given statement.



Exs. 2–7

2. If $AB = CD$ and $AX = CX$, then $AB - AX = CD - CX$.

3. If $\overrightarrow{XY}$ bisects $\angle AXD$, then $m\angle 2 = \frac{1}{2}m\angle AXD$.

4. If $\angle A \cong \angle C$ and $\angle C \cong \angle D$, then $\angle A \cong \angle D$.

5. If $AX = CX$ and $AX + XB = AB$, then $CX + XB = AB$.

6. $m\angle 1 = m\angle DXB$

7. Name two angles that are supplements of $\angle CXB$.

8. The measure of a supplement of an angle is four times as large as the measure of a complement of the angle. Find the measures of all three angles.

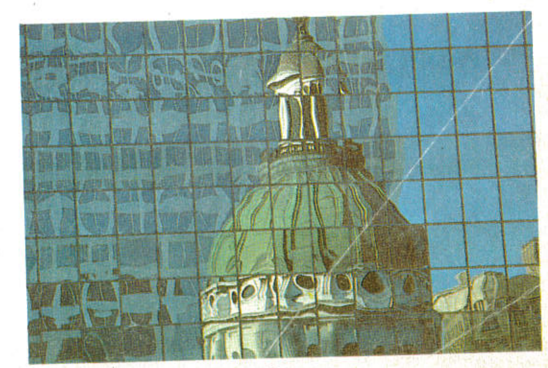
More about Proof

Objectives

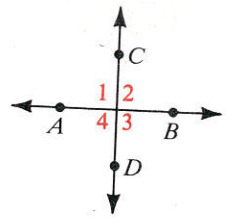
1. State and apply the theorems about perpendicular lines, supplementary angles, and complementary angles.
2. Recognize the information conveyed by a diagram.
3. Plan and write two-column proofs.
4. Understand the relationships described in the postulates and theorems of Section 1-9.

1-7 Perpendicular Lines

Since a supplement of a right angle is a right angle, you know that if two lines intersect to form one right angle, they actually form four right angles. The photo shows the reflection of a building in a grid of window panes. Notice that where any two of the grid lines intersect, four right angles are formed.



Perpendicular lines ($\perp$ lines) are two lines that form right angles. This definition can be used in the following situations.



1. If $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$ ($\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$), then each numbered angle is a right angle.
2. If any of the numbered angles is a right angle, then $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

The word "perpendicular" is also used for intersecting rays and segments that are parts of perpendicular lines. For example, if $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$ in the diagram, then we can also say that $\overline{CD} \perp \overline{AB}$.

The following theorems are easily deduced by using the definition of perpendicular lines. You will complete the proofs of the theorems in Classroom Exercise 10 and Written Exercises 9 and 10.

Theorem 1-4

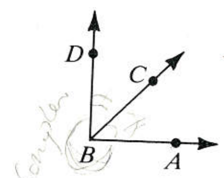
Adjacent angles formed by perpendicular lines are congruent.

Theorem 1-5

If two lines form congruent adjacent angles, then the lines are perpendicular.

Theorem 1-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

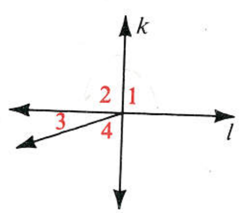


Given: $\overrightarrow{BA} \perp \overrightarrow{BD}$

Prove: $\angle ABC$ and $\angle CBD$ are comp. $\measuredangle$s.

The following example shows how these theorems can be used in a proof.

Example



Given: $\angle 1 \cong \angle 2$

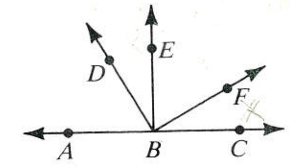
Prove: $\angle 3$ and $\angle 4$ are comp. $\measuredangle$s.

Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $k \perp l$	2. If 2 lines form $\cong$ adj. $\measuredangle$s, then the lines are $\perp$.
3. $\angle 3$ and $\angle 4$ are comp. $\measuredangle$s.	3. If the ext. sides of 2 adj. $\measuredangle$s are $\perp$, then the $\measuredangle$s are comp.

Classroom Exercises

In the diagram, $\overrightarrow{BE} \perp \overleftrightarrow{AC}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the measures of the following angles.

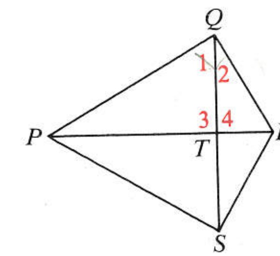


Exs. 1–8

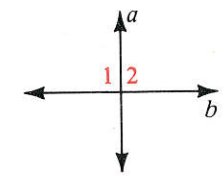
1. a. $\angle ABE$ **b.** $\angle DBF$

	$m\angle CBF$	$m\angle EBF$	$m\angle DBE$	$m\angle DBA$	$m\angle DBC$
2.	40	?	?	?	?
3.	x	?	?	?	?

State the definition or theorem that justifies the statement about the diagram.



4. If $\overrightarrow{QP} \perp \overrightarrow{QR}$, then $\angle 1$ and $\angle 2$ are complementary.
5. If $\overleftrightarrow{PR} \perp \overleftrightarrow{QS}$, then $\angle 4$ is a right angle.
6. If $\angle PSR$ is a right angle, then $\overleftrightarrow{PS} \perp \overleftrightarrow{SR}$.
7. If $\angle 3 \cong \angle 4$, then $\overline{PR} \perp \overline{QS}$.
8. If $\angle 3$ is a right angle, then $m\angle 3 = 90$.
9. If $m\angle PSR = 90$, then $\angle PSR$ is a right angle.
10. Complete the proof of Theorem 1-4: Adjacent angles formed by perpendicular lines are congruent.



Given: $a \perp b$

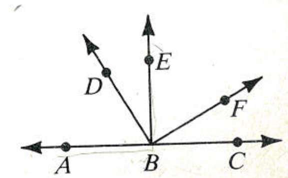
Prove: $m\angle 1 = m\angle 2$

Proof:

Statements	Reasons
1. $a \perp b$	1. __?__
2. $\angle 1$ and $\angle 2$ are right angles.	2. __?__
3. $m\angle 1 =$ __?__ and $m\angle 2 =$ __?__	3. __?__
4. __?__	4. __?__

Written Exercises

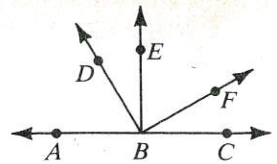
Write the definition or theorem that justifies the statement about the diagram.



A

1. If $\angle EBC$ is a right angle, then $\overrightarrow{BE} \perp \overleftrightarrow{AC}$.
2. If $\angle ABE$ is a right angle, then $m\angle ABE = 90$.
3. If $\overrightarrow{BE} \perp \overleftrightarrow{AC}$, then $\angle ABD$ and $\angle DBE$ are complementary.
4. If $\angle ABE \cong \angle CBE$, then $\overleftrightarrow{BE} \perp \overleftrightarrow{AC}$.

Write the definition or theorem that justifies the statement about the diagram.

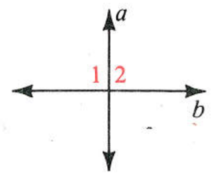


Exs. 1–8

5. If $m\angle DBF = 90$, then $\angle DBF$ is a right angle.
6. If $\overleftrightarrow{AC} \perp \overleftrightarrow{BE}$, then $m\angle ABE = m\angle CBE$.
7. If $\overleftrightarrow{AC} \perp \overleftrightarrow{BE}$, then $\angle ABE$ is a right angle.
8. If $\angle ABD$ and $\angle DBE$ are complements, then $m\angle ABD + m\angle DBE = 90$.

Copy and complete the proofs of Theorems 1-5 and 1-6.

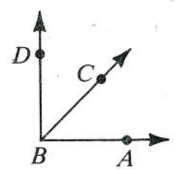
9. If two lines form congruent adjacent angles, then the lines are perpendicular.
   Given: $m\angle 1 = m\angle 2$
   Prove: $a \perp b$



Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180$	1. ?
2. $m\angle 1 = m\angle 2$	2. ?
3. $m\angle 1 + m\angle 1 = 180$, or $2m\angle 1 = 180$	3. ?
4. ?	4. Division Prop. of =
5. $\angle 1$ is a rt. $\angle$.	5. ?
6. ?	6. ?

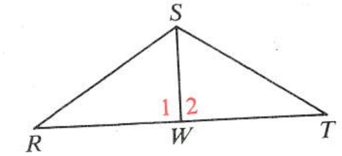
10. If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.
    Given: $\overrightarrow{BA} \perp \overrightarrow{BD}$
    Prove: $\angle ABC$ and $\angle CBD$ are comp. $\angle$s.



Proof:

Statements	Reasons
1. $\overrightarrow{BA} \perp \overrightarrow{BD}$	1. ?
2. ?	2. Def. of $\perp$ lines
3. $m\angle ABD = 90$	3. ?
4. $m\angle ABD = m\angle ABC + m\angle CBD$	4. ?
5. ?	5. Substitution Prop.
6. ?	6. Def. of comp. $\angle$s

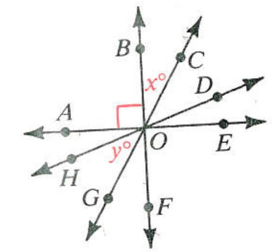
Copy everything shown and write a *short* two-column proof, using a theorem stated in this section.



B 11. Given: $\overline{SW} \perp \overline{RT}$
   Prove: $m\angle 1 = m\angle 2$

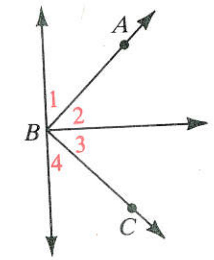
12. Given: $m\angle 1 = m\angle 2$
   Prove: $\overline{SW} \perp \overline{RT}$

In the figure, $\overleftrightarrow{BF} \perp \overleftrightarrow{AE}$, $m\angle BOC = x$, and $m\angle HOG = y$. Express the measure of the angle in terms of x, y, or both.



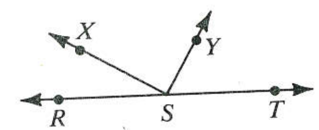
13. $\angle COA$
14. $\angle COH$
15. $\angle HOF$
16. $\angle DOE$

Can you conclude from the given information that $\overrightarrow{BA} \perp \overrightarrow{BC}$?

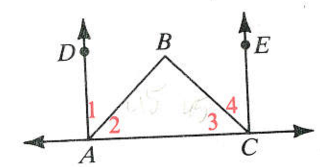


17. $m\angle 1 = 46$ and $m\angle 4 = 44$
18. $\angle 1$ and $\angle 3$ are complementary.
19. $\angle 2 \cong \angle 3$
20. $m\angle 1 = m\angle 4$
21. $\angle 1$ and $\angle 3$ are congruent and complementary.
22. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$
23. $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$
24. $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$

What can you conclude from the given information?

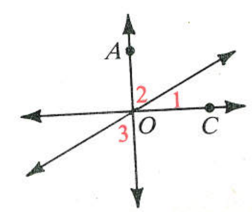


25. Given: $\overrightarrow{SX} \perp \overrightarrow{SY}$
26. Given: $\angle RSX$ and $\angle YST$ are comp. $\angle$s.



27. Given: $\overrightarrow{AB}$ bisects $\angle DAC$;
    $\overrightarrow{CB}$ bisects $\angle ECA$;
    $m\angle 2 = 45$;
    $m\angle 3 = 45$
28. Given: $\overrightarrow{AD} \perp \overleftrightarrow{AC}$; $\overrightarrow{CE} \perp \overleftrightarrow{AC}$; $m\angle 1 = m\angle 4$

Copy everything shown and write a two-column proof.



C 29. Given: $\angle 1$ and $\angle 2$ are comp. $\angle$s.
   Prove: $\overleftrightarrow{AO} \perp \overleftrightarrow{CO}$

30. Given: $\overleftrightarrow{AO} \perp \overleftrightarrow{CO}$
   Prove: $\angle 1$ and $\angle 3$ are comp. $\angle$s.