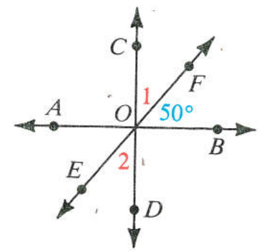
Classroom Exercises

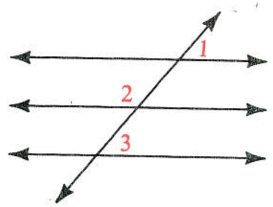
Given the figure, state whether you can reach the conclusion shown.

1. $m\angle FOB = 50$ 2. $m\angle AOC = 90$ 3. $m\angle DOC = 180$
4. $AO = OB$ 5. $\angle AOC \cong \angle BOC$ 6. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

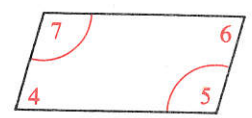


Exs. 1–12

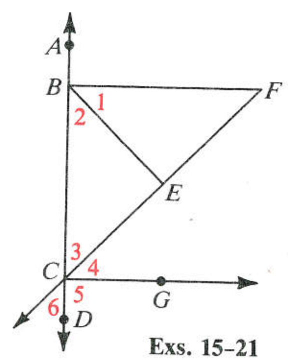
7. Points E, O, and F are collinear.
8. Point C is in the interior of $\angle AOF$.
9. $\angle AOE$ and $\angle AOD$ are adjacent angles.
10. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays.
11. O is between A and B.
12. $\angle 1$ and $\angle 2$ are vertical angles.
13. Given: $\angle 1$ is a supplement of $\angle 2$;
    $\angle 2$ is a supplement of $\angle 3$.
    State the theorem that allows you to conclude that $\angle 1 \cong \angle 3$.



14. Given: $\angle 4$ is a supplement of $\angle 5$;
    $\angle 6$ is a supplement of $\angle 7$;
    $\angle 5 \cong \angle 7$
    State the theorem that allows you to conclude that $\angle 4 \cong \angle 6$.



What can you deduce from the given information? State the definitions, postulates, and theorems that justify your deduction.

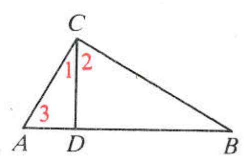


Exs. 15–21

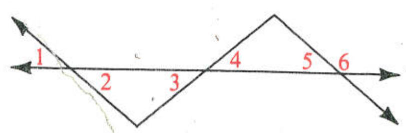
15. Given: $m\angle 1 = m\angle 4$; $m\angle 2 = m\angle 3$
16. Given: $AB = CD$
17. Given: $m\angle 6 = m\angle 4$
18. Given: $\overline{FB} \perp \overline{AD}$; $\overrightarrow{BE}$ bisects $\angle FBC$.
19. Given: $BE = EF$; E is the midpoint of $\overline{FC}$.
20. Given: $\angle 1$ and $\angle 2$ are complements.
21. Given: $\angle 1$ and $\angle 3$ are complements; $\overline{GC} \perp \overline{AD}$

Describe your plan for proving the following.

22. Given: $\overline{AC} \perp \overline{BC}$;
    $\angle 3$ is comp. to $\angle 1$,
    Prove: $\angle 3 \cong \angle 2$



23. Given: $\angle 2 \cong \angle 3$;
    $\angle 4 \cong \angle 5$
    Prove: $\angle 1$ is supp. to $\angle 6$.

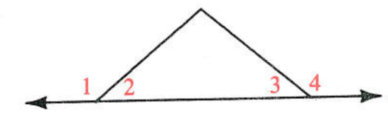


24. Refer to the proof of Theorem 1-7 on page 41 and consider the case when the two angles are supplementary to the same angle.
    a. Draw a figure, and state what is given and what is to be proved.
    b. Describe how you would change the proof on page 41 to prove the conclusion in part (a).

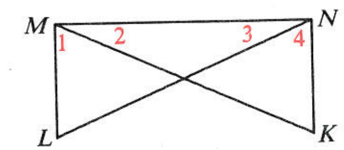
Written Exercises

A

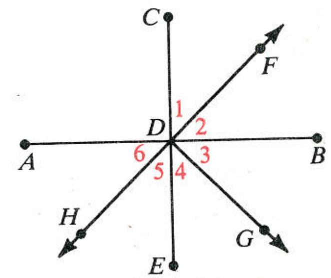
1. a. Name a supplement of $\angle 2$.
   b. Name a supplement of $\angle 3$.
   c. What postulate or theorem, along with the definition of supplementary angles, justifies your answers to parts (a) and (b)?
   d. If $\angle 2 \cong \angle 3$, write the theorem that allows you to conclude that $\angle 1 \cong \angle 4$.



2. In the diagram, $\overline{LM} \perp \overline{MN}$ and $\overline{KN} \perp \overline{MN}$.
   a. Name a complement of $\angle 2$.
   b. Name a complement of $\angle 3$.
   c. Write the theorem that justifies your answers to parts (a) and (b).
   d. If $\angle 2 \cong \angle 3$, write the theorem that allows you to conclude that $\angle 1 \cong \angle 4$.



Write the name or statement of the definition, postulate, property, or theorem that justifies the statement about the diagram.



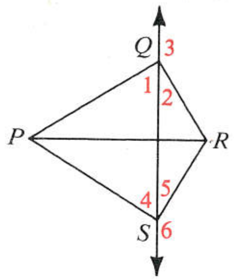
Exs. 3–18

3. $AD + DB = AB$
4. $m\angle 1 + m\angle 2 = m\angle CDB$
5. If $AD = DB$ and $CD = DE$, then $AD + CD = DB + DE$.
6. $\angle 2 \cong \angle 6$
7. If $\overrightarrow{DF}$ bisects $\angle CDB$, then $m\angle 1 = m\angle 2$.
8. If D is the midpoint of $\overline{AB}$, then $AD = DB$.
9. If $\overline{CD} \perp \overline{AB}$, then $m\angle CDB = 90$.
10. $m\angle ADF + m\angle FDB = 180$
11. If $m\angle 3 + m\angle 4 = 90$, then $\angle 3$ and $\angle 4$ are complements.
12. If $\angle ADF$ and $\angle 4$ are supplements, then $m\angle ADF + m\angle 4 = 180$.
13. If D is the midpoint of $\overline{CE}$, then $CE = 2 \cdot DE$.
14. If $m\angle 4 = m\angle 3$, then $\overrightarrow{DG}$ bisects $\angle BDE$.
15. If $\overline{AB} \perp \overline{CE}$, then $\angle ADC \cong \angle ADE$.
16. If $\angle 4$ is complementary to $\angle 5$ and $\angle 6$ is complementary to $\angle 5$, then $\angle 4 \cong \angle 6$.
17. If $m\angle FDG = 90$, then $\overrightarrow{DF} \perp \overrightarrow{DG}$.
18. If $m\angle FDG = m\angle GDH$, then $\overrightarrow{DG} \perp \overleftrightarrow{HF}$.

19. a. Complete the proof.

Given: $\overline{PQ} \perp \overline{QR}$;
$\overline{PS} \perp \overline{SR}$;
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 5$



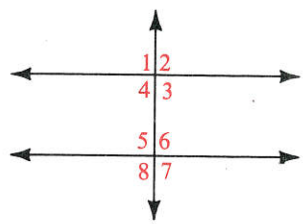
Proof:

Statements	Reasons
1. $\overline{PQ} \perp \overline{QR}$; $\overline{PS} \perp \overline{SR}$	1. ?
2. $\angle 2$ is comp. to $\angle 1$; $\angle 5$ is comp. to $\angle 4$.	2. ?
3. $\angle 1 \cong \angle 4$	3. ?
4. $\angle 2 \cong \angle 5$	4. ?

b. Now that you have proved that $\angle 2 \cong \angle 5$, describe a plan for proving that $\angle 3 \cong \angle 6$.

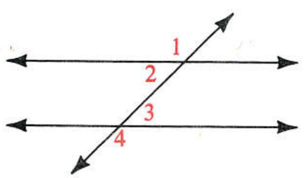
20. a. Are there any angles in the diagram that must be congruent to $\angle 4$? Explain.

b. If $\angle 4$ and $\angle 5$ are supplementary, name all angles shown that must be congruent to $\angle 4$.

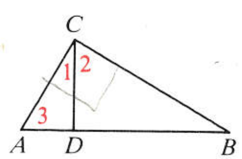


Copy everything shown and write a two-column proof.

B **21.** Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



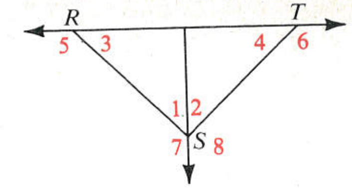
22. Given: $\overline{AC} \perp \overline{BC}$
$\angle 3$ is comp. to $\angle 1$.
Prove: $\angle 3 \cong \angle 2$



23. Prove Theorem 1-8: If two angles are complements of congruent angles, then the two angles are congruent. (*Hint:* See the proof of Theorem 1-7 on page 41.)

24. Write a two-column proof of Theorem 1-3 that is different from the one on page 31.

25. Given: $\overline{RS} \perp \overline{ST}$;
$\angle 1$ and $\angle 4$ are comp. $\angle$s;
$\angle 2$ and $\angle 3$ are comp. $\angle$s.

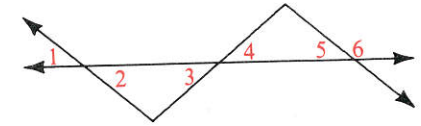


Complete:

a. $\angle 1 \cong \angle$? **b.** $\angle 2 \cong \angle$?
c. $\angle 5 \cong \angle$? **d.** $\angle 6 \cong \angle$?

Copy everything shown and write a two-column proof.

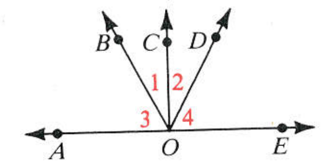
26. Given: $\angle 4$ is supp. to $\angle 6$.
Prove: $\angle 3 \cong \angle 5$



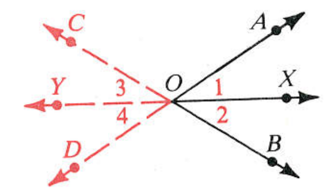
27. Given: $\angle 2 \cong \angle 3$;
$\angle 4 \cong \angle 5$
Prove: $\angle 1$ is supp. to $\angle 6$.

28. Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$
Prove: $\overrightarrow{OC} \perp \overleftrightarrow{AE}$

29. Given: $\overrightarrow{OC} \perp \overleftrightarrow{AE}$; $\overrightarrow{OC}$ bisects $\angle BOD$.
Prove: $m\angle 3 = m\angle 4$

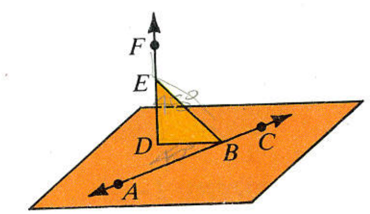


C **30.** Draw any $\angle AOB$ and its bisector $\overrightarrow{OX}$. Now draw the rays opposite to $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OX}$. What can you conclude? Prove it.



31. Make a diagram showing $\angle PQR$ bisected by $\overrightarrow{QX}$. Choose a point Y on the ray opposite $\overrightarrow{QX}$.
Prove: $\angle PQY \cong \angle RQY$

32. Given: $m\angle DBA = 45$;
$m\angle DEB = 45$
Prove: $\angle DBC \cong \angle FEB$



1-9 Postulates Relating Points, Lines, and Planes

Our first postulates and theorems have dealt primarily with segments and their lengths and with angles and their measures. We will be able to prove many more things about geometric figures after we make the following basic assumptions about relationships between points, lines, and planes.