Written Exercises

A 1. Two cones have radii 6 and 9. The heights are 10 and 15. Are the cones similar?

2. The heights of two right prisms are 18 and 30. The bases are squares with sides 8 and 15. Are the prisms similar?

3. Two similar cylinders have radii 3 and 4. Find the ratios of the following:
   a. heights   **b.** base circumferences   **c.** lateral areas   **d.** volumes

4. Two similar pyramids have heights 12 and 18. Find the ratios of the following:
   a. base areas   **b.** lateral areas   **c.** total areas   **d.** volumes

5. Assume that the Earth and the moon are smooth spheres with diameters 12,800 km and 3,200 km, respectively. Find the ratios of the following:
   a. lengths of their equators   **b.** areas   **c.** volumes

6. The package of a model airplane kit states that the scale is 1:200. Compare the amounts of paint required to cover the model and the actual airplane. (Assume the paint on the model is as thick as that on the actual plane.)

7. The scale for a certain model freight train is 1:48. If the model hopper car (usually used for carrying coal) will hold 90 in.^{3} of coal, what is the capacity of the actual hopper car?

8. Two similar cylinders have lateral areas 81π and 144π. Find the ratios of the following:
   **a.** heights   **b.** total areas   **c.** volumes

9. Two similar cones have volumes 8π and 27π. Find the ratios of the following:
   a. radii   **b.** slant heights   **c.** lateral areas

10. Two similar pyramids have volumes 3 and 375. Find the ratios of the following:
   **a.** heights   **b.** base areas   **c.** total areas

B 11. A certain kind of string is sold in a ball 6 cm in diameter and in a ball 12 cm in diameter. The smaller ball costs $1.00 and the larger one costs $6.50. Which is the better buy?

12. Two balls made of the same metal have radii 6 cm and 10 cm. If the smaller ball weighs 4 kg, how much does the larger ball weigh?

13. A snow man is made using three balls of snow with diameters 30 cm, 40 cm, and 50 cm. If the head weighs roughly 6 kg, find the total weight of the snow man. (Ignore arms, eyes, nose, and mouth.)
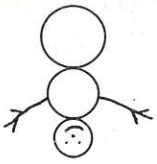
14. Construction engineers know that the strength of a column is proportional to the area of its cross section. Suppose that the larger of two similar columns is three times as high as the smaller column.
    a. The larger column is __?__ times as strong as the smaller column.
    b. The larger column is __?__ times as heavy as the smaller column.
    c. Which can support more, *per pound of column material,* the larger or the smaller column?
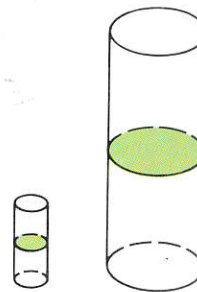
15. Two similar pyramids have lateral areas 8 and 18. If the volume of the smaller pyramid is 32, what is the volume of the larger?

16. Two similar cones have volumes 12π and 96π. If the lateral area of the smaller cone is 15π, what is the lateral area of the larger?

17. A plane parallel to the base of a cone divides the cone into two pieces. Find the ratios of the following:
    a. The areas of the shaded circles
    b. The lateral area of the top piece to that of the whole cone
    c. The lateral area of the top piece to that of the bottom piece
    d. The volume of the top piece to that of the whole cone
    e. The volume of the top piece to that of the bottom piece

18. Redraw the figure for Exercise 17, changing the 9 cm and 3 cm dimensions to 10 cm and 4 cm. Then find the five ratios described in Exercise 17.

19. A pyramid with height 15 cm is separated into two pieces by a plane parallel to the base and 6 cm above it. What are the volumes of these two pieces if the volume of the original pyramid is 250 cm³?
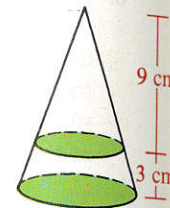
The purpose of Exercises 20–25 is to prove Theorem 10-3 for some similar solids.

20. Two spheres have radii a and b. Prove that the ratio of the areas is $a^2:b^2$.

21. Two spheres have radii a and b. Prove that the ratio of the volumes is $a^3:b^3$.

22. Two similar cones have radii r_1 and r_2 and heights h_1 and h_2. Prove that the ratio of the volumes is $h_1^3:h_2^3$.

23. Two similar cones have radii r_1 and r_2 and lateral heights l_1 and l_2. Prove that the ratio of the lateral areas is $r_1^2:r_2^2$.

24. The bases of two similar prisms are regular pentagons with base edges e_1 and e_2 and base areas B_1 and B_2. The heights are h_1 and h_2. Prove that the ratio of the lateral areas is $e_1^2:e_2^2$.

25. Refer to Exercise 24 and prove that the ratio of the volumes of the prisms is $e_1^3:e_2^3$.