Some true statements have false converses.

True statement: If a figure is a triangle, then it is a polygon.
False converse: If a figure is a polygon, then it is a triangle.

Some true statements have true converses.

True statement: If a triangle is equilateral, then it is equiangular.
True converse: If a triangle is equiangular, then it is equilateral.

When a statement and its converse are both true, we can combine them into one statement by using the words "if and only if" as follows:

A triangle is equilateral *if and only if* it is equiangular.

This means that:

1. A triangle is equilateral if it is equiangular.
2. A triangle is equilateral only if it is equiangular.

Similarly the statement

p if and only if q

means:

1. p if q. (Equivalent to "If q, then p.")
2. p only if q. (Equivalent to "If p, then q.")

All the definitions you have learned could have been stated as if-and-only-if statements. For example, the definition *Perpendicular lines are two lines that form right angles* tells us two things:

1. If two lines are perpendicular, then the lines form right angles.
2. If two lines form right angles, then the lines are perpendicular.

Thus, the definition could have been stated in this form:

Two lines are perpendicular if and only if the lines form right angles.

Classroom Exercises

State the hypothesis and conclusion of each statement.

1. If $2x - 1 = 5$, then $x = 3$.
2. If I'm smart, then she's a genius.
3. $x \div (-2) = 9$ implies that $x = -18$.
4. I'll go if you go.
5. There is smoke only if there is fire.
6. You can if you try.
7. You can only if you try.

Give the converse of the statement in the exercise listed.

8. Exercise 1 **9.** Exercise 4 **10.** Exercise 5

Express each statement in if-then form.

11. All equilateral triangles are isosceles. (Begin "If a figure is . . .")
12. The diagonals of a rectangle are congruent.
13. All students love vacations. (Begin "If a person is . . .")
14. Every rectangle is equiangular.
15. He will drive provided it doesn't snow.
16. $x > -2$ whenever $-4x < 8$.
17. Give an example of a true statement that has a false converse.
18. Give an example of a true statement that has a true converse.

Replace each statement by two if-then statements that are converses of each other.

19. A triangle is obtuse if and only if it contains an obtuse angle.
20. Two angles are complementary if and only if the sum of their measures is 90.

Written Exercises

For each statement, state (a) the hypothesis, (b) the conclusion, and (c) the converse.

A **1.** If $3x - 7 = 32$, then $x = 13$.
2. If $\overline{AB} \perp \overline{BC}$, then $m\angle ABC = 90$.
3. I'll try if you will.
4. I can't sleep if I'm not tired.
5. $|x| = 0$ only if $x = 0$.
6. $a + b = a$ implies $b = 0$.

For each statement, (a) rewrite the statement in if-then form and state whether it is true or false. Then (b) write the converse and state whether it is true or false.

7. All Olympic competitors are athletes.
8. Every positive number has two square roots.
9. $x^2 = 0$ only if $x = 0$.
10. $x^2 = 16$ when $x = 4$.
11. The product of two odd integers is odd.
12. The sum of two even integers is even.
13. $-2x < 2$ implies $x > -1$.
14. $\triangle XYZ$ is obtuse only if $\angle XYZ$ is obtuse.
15. Every regular polygon is equiangular.
16. All even integers are divisible by 4.

Rewrite each statement as two if-then statements that are converses of each other.

17. Two angles are congruent angles if and only if their measures are equal.
18. Two angles are supplementary if and only if the sum of their measures is 180.
19. $ab > 0$ if and only if a and b are both positive or both negative.
20. $(x - 4)(x + 6) = 0$ if and only if $x = 4$ or $x = -6$.
21. Postulate 10 may also be worded in this way: "If two lines are parallel, then corresponding angles formed by the two lines and a transversal are congruent." Write the converse of this statement. Is the converse true?
22. Theorem 2-5 may also be worded in this way: "If alternate interior angles formed by two lines and a transversal are congruent, then the lines are parallel." Write the converse of this statement. Is the converse true?
23. **a.** Write the converse of the Subtraction Property of Equality: If $a = b$ and $c = d$, then $a - c = b - d$.
    b. Choose values of a, b, c, and d to show that the converse is false.

Tell whether you think each statement is true or false.
If false, draw a diagram that shows a counterexample.
If true, draw a diagram. List, in terms of the diagram, what is given and what is to be proved. Do *not* write a proof.

B 24. If a triangle has two congruent sides, then the angles opposite those sides are congruent.
25. If a triangle has two congruent angles, then the sides opposite those angles are congruent.
26. Two triangles have equal perimeters only if they have congruent sides.
27. All diagonals of a regular pentagon are congruent.
28. If both pairs of opposite sides of a quadrilateral are parallel, then each side is congruent to the side opposite it.
29. If the diagonals of a quadrilateral are congruent and also perpendicular, then the quadrilateral is a regular quadrilateral.
30. The diagonals of an equilateral quadrilateral are congruent.
31. The diagonals of an equilateral quadrilateral are perpendicular.

In the conditional statement "if p, then q," p is said to be a *sufficient* condition for q to occur. Similarly, q is said to be a *necessary* condition for p. In Exercises 32–37 tell whether the first statement is necessary for the second statement, sufficient for it, or both necessary and sufficient for it.

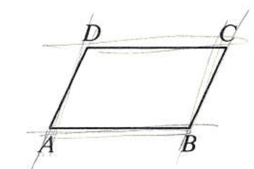
Example

	First statement	*Second statement*
a.	An integer is divisible by 2.	An integer is even.
b.	Lines l and m are coplanar.	Lines l and m are parallel.

Solution **a.** necessary and sufficient (An integer is divisible by 2 if and only if the integer is even.)
b. necessary (If lines l and m are parallel, then they are coplanar. Note that the first statement is not sufficient for the second because two lines may be coplanar without being parallel.)

	First statement	*Second statement*
C 32.	$x > 4$	x is positive.
33.	An integer is odd.	The square of an integer is odd.
34.	Lines l and m do not intersect.	Lines l and m are parallel.
35.	$\angle A$ is a right angle.	$\triangle ABC$ is a right triangle.
36.	A polygon is equilateral.	A polygon is regular.
37.	Alternate interior angles formed by lines l and m and transversal t are congruent.	Lines l and m are parallel.

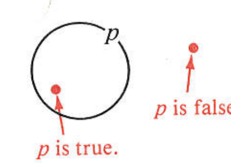
38. **a.** Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$
    Prove: $\angle A \cong \angle C$; $\angle B \cong \angle D$
    b. Tell what is given and what is to be proved in the converse of part (a). Then write a proof of the converse.
    c. Combine what you have proved in parts (a) and (b) into an if-and-only-if statement.



2-7 Converse, Contrapositive, Inverse

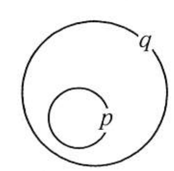
To show the relationship between an if-then statement and its converse, it is helpful to use circle diagrams (also called Venn diagrams or Euler diagrams).

To represent a statement p, we draw a circle named p. If p is true, we think of a point inside circle p. If p is false, we think of a point outside circle p.

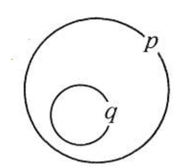


In the diagram at the left below, a point that lies inside circle p must also lie inside circle q. In other words: *If p, then q.* Check to see that the middle diagram represents the converse: *If q, then p.* Check the diagram at the right also.

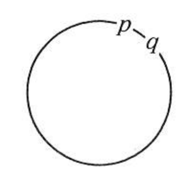
P only if Q
IF P, THEN Q



If p, then q.

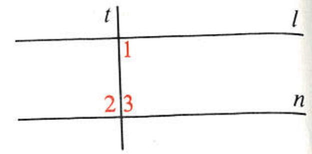


If q, then p.



p if and only if q.

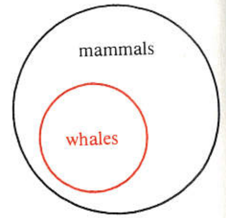
Classify each statement as true or false. Then classify its contrapositive, converse, and inverse as true or false.



Exs. 7–9

6. If $x = -5$, then $x^2 = 25$.
7. If $\angle 1 \cong \angle 2$, then $l \parallel n$.
8. If $l \parallel n$, then t is perpendicular to both l and n.
9. If l is not parallel to n, then $m\angle 1 + m\angle 3 \neq 180$.

Given statement: All whales are mammals.



Exs. 11–13

10. Reword the statement in if-then form.
11. Explain how the diagram represents both the statement and its contrapositive.
12. Accept the statement below as true and pair it with the given statement. What can you conclude? If no conclusion is possible, say so.
    a. Moby is a whale.
    b. Mabel is not a whale.
    c. Manfred is a mammal.
    d. Myrtle is not a mammal.
13. Copy the diagram on the chalkboard. Locate points to represent statements (a)-(d) in Exercise 12. Use the points to check your conclusions in Exercise 12.

Accept the first two statements as true. Does the third statement follow as a conclusion?

14. An elephant never forgets.
    Ed is not an elephant.
    Thus, Ed sometimes forgets.
15. An elephant never forgets.
    Ellie never forgets.
    Thus, Ellie is an elephant.
16. An elephant never forgets.
    Edith sometimes forgets.
    Thus, Edith is not an elephant.

Written Exercises

State (a) the contrapositive, (b) the converse, and (c) the inverse of each statement.

A

1. If $n = 21$, then $5n - 5 = 100$.
2. If that is red, then this is white.
3. If x is not even, then $x + 1$ is not odd.
4. If Gregory is not here, then he is not well.

Make a circle diagram to illustrate each statement.

5. If the car has airbags, then it must be new.
6. A triangle is equilateral if and only if it is equiangular.
7. All mice like cheese.
8. No musician likes noise.

For each statement in Exercises 9–15, copy and complete a table like the one shown below.

		True/False?
Statement	?	?
Contrapositive	?	?
Converse	?	?
Inverse	?	?

9. If $\angle 1 \cong \angle 2$, then $\angle 1$ and $\angle 2$ are vertical angles.
10. If $x > 6$, then $x = 5$.

B

11. If $AM = MB$, then M is the midpoint of $\overline{AB}$.
12. If $x^2 - 1 = 99$, then $x = 10$.
13. If a is not negative, then $|a| = a$.
14. If $-3a > -6$, then $a < 2$.
15. If $x^2 > y^2$, then $x > y$.
16. Given: All senators are at least 30 years old.
    a. Reword this statement in if-then form.
    b. Make a circle diagram to illustrate the statement.
    c. If the given statement is true, what can you conclude from each of the following additional statements? If no conclusion is possible, say so.
       (1) Jose Avila is 48 years old.
       (2) Rebecca Castelloe is a senator.
       (3) Constance Brown is not a senator.
       (4) Ling Chen is 29 years old.

In Exercises 17–21, assume that the given statement is true. Then tell what you can conclude if each statement in (a)-(d) is also true. If no conclusion is possible, say so.

17. Given: If a quadrilateral is equiangular, then its diagonals are congruent.
    a. $ABCD$ is equiangular.
    b. $PQRS$ has congruent diagonals.
    c. In $WXYZ$, $m\angle X = 80$.
    d. In $EFGH$, $EG > FH$.
18. Given: If it is not raining, then I am happy.
    a. I am not happy.
    b. It is not raining.
    c. I am overjoyed.
    d. It is raining.
19. Given: All my students love geometry.
    a. Stu is my student.
    b. Luis loves geometry.
    c. Stella is not my student.
    d. George does not love geometry.