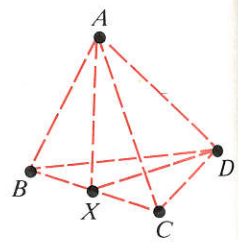
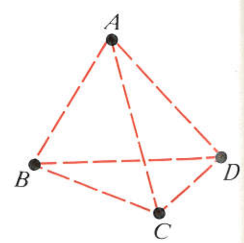
Postulate 5 requires space to contain *at least* four points not all in one plane and any line to contain *at least* two points. Geometry A works with exactly four points, and we have developed Geometry B using just four points (persons). But a finite geometry satisfying Postulates 5–9 can have more than four points, and more than two points on a line. Some of the exercises below explore finite geometries with more than four points.

However, no finite geometry can satisfy all the postulates of this book. Consider Postulate 1, page 7. That postulate calls for infinitely many points, so you see that the finite geometry shown in Geometry A does not satisfy all the postulates of this book. How about Geometry B? Since the number of living persons is finite, you know that we cannot use the interpretation discussed in Geometry B when all the postulates of the book are to be satisfied.

Exercises



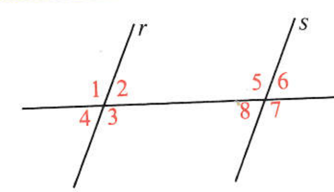
1. Refer to Geometry B and restate Postulates 5–9 in terms of *person, committee,* and *club*. Also restate Theorems 1-9, 1-10, and 1-11.
2. **a.** Check the five-point geometry shown to decide whether Postulates 5–9 are satisfied.
   b. Apply the Geometry B interpretation to the diagram. For the fifth point X, use a fifth person, Xavera. List the members of a three-person committee and the members of two four-member clubs.
3. Limiting yourself to Postulates 5–9, write the key steps of a proof of the theorem: If there are five points not all in one plane, then there are at least eight lines and five planes. (*Hint:* Let A, B, C, and D be four points not all in one plane. Consider three cases for a fifth point X.
   (1) X lies on one of the lines—use (BC)—determined by A, B, C, D.
   (2) X lies on one of the planes, but not on any line, determined by A, B, C, D.
   (3) X does not lie on any plane determined by A, B, C, D.
4. Refer to Exercise 3. State the theorem in terms of persons. Note that no further proof is necessary.
5. Suppose a finite geometry satisfying Postulates 5–9 contains exactly six points.
   a. What is the largest possible number of different lines?
   b. What is the smallest possible number of different lines?
   c. What is the largest possible number of different planes?
   d. What is the smallest possible number of different planes?
6. For this exercise, we define two lines to be parallel when they do not have any point in common. Consider the five-point geometry pictured above and Geometry A, which is pictured again at the right. Decide whether each of these geometries satisfies the following statement: Through a point outside a line, there is exactly one line parallel to the given line.



Chapter Summary

1. Lines that do not intersect are either parallel or skew.
2. When two parallel lines are cut by a transversal:
   a. corresponding angles are congruent;
   b. alternate interior angles are congruent;
   c. same-side interior angles are supplementary;
   d. if the transversal is perpendicular to one of the two parallel lines, it is also perpendicular to the other one.
3. The chart on page 67 lists five ways to prove lines parallel.
4. Through a point outside a line, there is exactly one line parallel to, and exactly one line perpendicular to, the given line.
5. Two lines parallel to a third line are parallel to each other.
6. Triangles are classified (page 72) by the lengths of their sides and by the measures of their angles. In any $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180$.
7. The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.
8. The sum of the measures of the angles of a convex polygon with n sides is $(n - 2)180$. The sum of the measures of the exterior angles, one angle at each vertex, is 360.
9. The summary on page 92 gives the relationships between an if-then statement and its converse, its contrapositive, and its inverse. An if-then statement and its contrapositive are logically equivalent.

Chapter Review



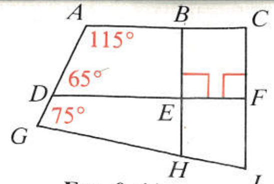
Exs. 1–7

1. $\angle 5$ and $\angle$ _?_ are same-side interior angles.
2. $\angle 5$ and $\angle 1$ are _?_ angles.
3. $\angle 5$ and $\angle 3$ are _?_ angles.
4. Line j, not shown, does not intersect line r. Must lines r and j be parallel?

In the diagram above, $r \parallel s$.

5. If $m\angle 1 = 105$, then $m\angle 5 =$ _?_ and $m\angle 7 =$ _?_.
6. Solve for x: $m\angle 2 = 70$ and $m\angle 8 = 6x - 2$
7. Solve for y: $m\angle 3 = 8y - 40$ and $m\angle 8 = 2y + 20$
8. Lines a, b, and c are coplanar, $a \parallel b$, and $a \perp c$. What can you conclude? Explain.

9. Which line is parallel to $\overleftrightarrow{AB}$? Why? 2-3

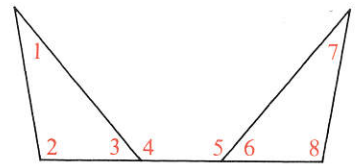


Exs. 9–11

10. Name a pair of parallel lines other than the pair in Exercise 9. Why must they be parallel?
11. Find the measure of $\angle I$.
12. Name five ways to prove two lines parallel.

13. If x and $2x - 15$ represent the measures of the acute angles of a right triangle, find the value of x. 2-4

14. $m\angle 6 + m\angle 7 + m\angle 8 = \underline{\ ?\ }$



Exs. 14–16

15. If $m\angle 1 = 30$ and $m\angle 4 = 130$, then $m\angle 2 = \underline{\ ?\ }$.
16. If $\angle 4 \cong \angle 5$ and $\angle 1 \cong \angle 7$, name two other pairs of congruent angles and give a reason for each answer.

17. a. Sketch a hexagon that is equiangular but not equilateral. 2-5
    b. What is its interior angle sum?
    c. What is its exterior angle sum?
18. A regular polygon has 18 sides. Find the measure of each interior angle.
19. A regular polygon has 24 sides. Find the measure of each exterior angle.
20. Each interior angle of a regular polygon has measure 156. How many sides does the polygon have?

Consider the statement "A quadrilateral is equilateral if it is a square."

21. Write the statement in if-then form. 2-6
22. Name the hypothesis and the conclusion.
23. Write the converse and state whether it is true or false.
24. Rewrite the following statement as two if-then statements that are converses of each other: Two segments are congruent if and only if their lengths are equal.

You are given the true statement "Toads are amphibians." In each exercise, accept the additional information as also true. What can you conclude, if anything?

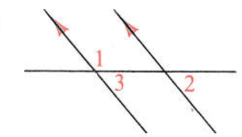
25. Toddie is a toad.
26. A frog is an amphibian. 2-7
27. A dog isn't an amphibian.
28. A tortoise isn't a toad.

Chapter Test

Complete each statement with the word *always, sometimes,* or *never*.

1. Two lines that have no points in common are $\underline{\ ?\ }$ parallel.
2. If a line is perpendicular to one of two parallel lines, then it is $\underline{\ ?\ }$ also perpendicular to the other one.
3. If two lines are cut by a transversal and same-side interior angles are complementary, then the lines are $\underline{\ ?\ }$ parallel.
4. An obtuse triangle is $\underline{\ ?\ }$ a right triangle.
5. In $\triangle ABC$, if $\overline{AB} \perp \overline{BC}$, then $\overline{AC}$ is $\underline{\ ?\ }$ perpendicular to $\overline{BC}$.
6. As the number of sides of a regular polygon increases, the measure of each exterior angle $\underline{\ ?\ }$ decreases.
7. The converse of a true if-then statement is $\underline{\ ?\ }$ true.

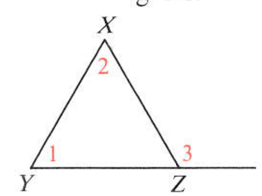
Find the value of x.



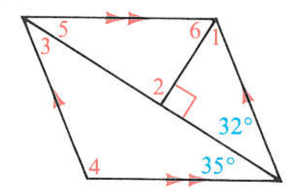
8. $m\angle 1 = 3x - 20$, $m\angle 2 = x$
9. $m\angle 2 = 2x + 12$, $m\angle 3 = 4(x - 7)$

Find the measures of the numbered angles.

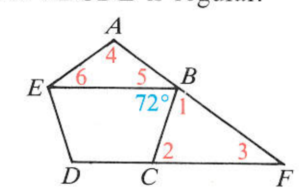
10. XYZ is regular.



11.



12. $ABCDE$ is regular.



13. In the diagram for Exercise 12, explain why $\overline{EB}$ and $\overline{DF}$ must be parallel.

Use the statement "If $x = y$, then $x^2 = y^2$."

14. Write the: **a.** hypothesis **b.** conclusion **c.** converse **d.** contrapositive
15. Pair each statement below with the given statement above and tell what conclusion, *if any,* must follow.
    a. $x^2 = y^2$ **b.** $x^2 \neq y^2$ **c.** $x = y$ **d.** $x \neq y$

16. Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overrightarrow{BF}$ bisects $\angle ABE$; $\overrightarrow{DG}$ bisects $\angle CDB$.
    Prove: $\overleftrightarrow{BF} \parallel \overleftrightarrow{DG}$